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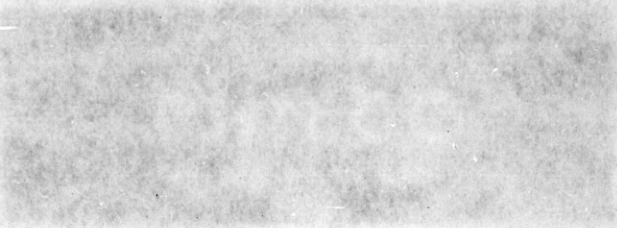

Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 238 OCTOBER, 1944 No. 4

"ORDNANCE RESEARCH AND DEVELOPMENT FOR PEACE."

BY

MAJOR GENERAL LEVIN H. CAMPBELL, JR.,

Chief of Ordnance, U. S. Army, War Department, Washington, D. C.

Mr. Redding, Dr. Allen, members of The Franklin Institute:

To stand in this historic place and to confer honor upon The Franklin Institute is a privilege of which I consider myself entirely unworthy. The seriousness of the times, the heroism of our soldiers, the patriotism of our fellow citizens, and the venerable record of this Institute—all combine to emphasize the seriousness of the occasion. I am deeply gratified for the opportunity you have afforded me to present the Ordnance Distinguished Service Citation to The Franklin Institute under such auspices. In making the presentation I have chosen to speak to you briefly on "Ordnance Research and Development For Peace."

Before addressing myself to this topic, first allow me to speak in a personal vein. I have great respect and appreciation of long standing for The Franklin Institute as an influence in the cultural progress of our country. This arises not only from the general reputation of the Institute as a technical and scientific organization but, even more so, because I have lived here in Philadelphia and have known personally many of its members. During my military service I was fortunate enough to be stationed at the Frankford Arsenal where there were many occasions to form the friendships with your officers and members which have continued for many years. And when I was not a resident of Philadelphia, during my tours of duty at other Ordnance establishments, I have had official contacts with the executives of the Institute

* Presented at the Special Meeting of The Franklin Institute held Wednesday, September 13, 1944.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

and with their research engineers, all of which gave me a keen appreciation of the scientific standing of your renowned Institute.

The reason for this presentation today is very simple. It is an attempt on the part of the Ordnance Department of the Army to acknowledge the cooperation which has existed over many years between our two organizations. The Ordnance Department of the Army itself has a long tradition of armament development dating back to 1812. At no time could these two institutions have cooperated in a more direct and fruitful way than has been the case during the past 25 years. The officials of the Institute, and Dr. Allen in particular, have been of inestimable help in our work for industrial mobilization planning for war. The Institute and the Philadelphia Ordnance District, of which Mr. Jared Ingersoll, one of your members, is Chief, have the closest ties of mutually helpful interest. Naturally enough when the present emergency arose we turned to the Institute and requested it to carry on certain researches for the improvement of our weapons. These responsibilities the Institute has carried forward with alacrity, thoroughness, and typical skill. The exact nature of these armament research projects I am not at liberty to disclose at this time. Later on when the battle is won and the armament research record can be publicly narrated for the edification of our fellow citizens, then these details will become public knowledge. Until then let it suffice that the scientists of Franklin Institute have waged the battle of research in a manner akin to the heroic deeds of our fighting men overseas. And were it not for this type of research the cost to our fighters on land, aloft and on the sea would be much greater. I assure you that your scientific prowess well deserves whatever acclaim we of the Ordnance Department can give you.

I shall not speak at length today because I do not want to detract one iota from the nature of this gathering. We are here to pay honor to whom honor is due. But the occasion is so fraught with significance for the security of our country in years to come that I want to record in this presence a plea for Ordnance research and development for peace.

Let me explain briefly why this subject deserves serious consideration now.

That part of the Army of the United States which is responsible for the engineering, design, development, and improvement of our weapons is the Ordnance Department. This is a traditional responsibility which has existed since the days of Decius Wadsworth, first Chief of Army Ordnance in 1812. Armament development, as carried on by the Ordnance Department at its arsenals and proving grounds and in conjunction with the technical and scientific organizations of our country, has been responsible not only for our superiority of weapons in time of war, but for many of our advancements to better living in

time of peace. It was the influence of the Ordnance Department and the handiwork of the New England gun makers early in the 19th century which brought to rifle design and production the principle of interchangeability of parts. That principle, borne of the musket, carbine and the shoulder rifle, is today responsible for the mass production through which American industry has attained the very pinnacle of higher quantity output with lower unit cost. Our industrial supremacy today rests firmly upon the doctrine of interchangeable parts which Eli Whitney, working in conjunction with Army Ordnance, developed decades ago.

Again, metals progress in our country and throughout the world owes much of its speed and thoroughness, to say nothing of its applicability in modern life, to Ordnance researches in cannon and projectile making. Gun making in the United States pioneered in the field of new and stronger metals, so that at the close of the 19th century vast industrial fields were opened up in the realms of transportation and of construction due to the impetus of the armsmaker aided and guided by the metals scientist and technologist.

So I might recount in many specific ways the cooperative endeavor between science and Ordnance throughout the years. Equally might I narrate the general advances that have come to all our people as by-products of this cooperative endeavor. War utilizes all the things of peace and improves them at a tempo so quickened that within the past four years scientific advancements have been made which might otherwise have required generations. Conversely peace utilizes the advancements of war. Despite its suffering, its cost in lives and money, and its sorrows, there are scientific gains from this conflict which generations of men will bless until the end of time.

The pity is that much of this progress has been wrought under great danger when hazards were such that our national existence was at stake. We must never forget the lesson we have learned since Pearl Harbor, and in no field is that lesson more important than in the need for constant research and development as applied to weapons for our national security.

The record of weapon research and development until 1940 is not a glorious one. The Ordnance Department of the Army, responsible for the design and development of our fighting equipment, did not receive in time of peace funds sufficient to carry forward research on a scale equal to any of our comparable industrial organizations. It was a great day for weapons' progress in our country ten years ago when as much—or as little—as $7,000,000 was appropriated in one year, not for research and development mind you, but for the operation of the entire Ordnance Department of the Army—its arsenals, depots and proving grounds, and yet when war came that responsibility rose in

money value to more than fifty billion dollars in one year. Those figures speak for themselves, and I will not attempt to elaborate on them. I will only record the firm belief that our weapons' superiority —and therefore our national security—in the years to come will be in direct ratio to the time, energy and money we allot to armament research and development. Adequate funds will be a small premium to allot to this form of national peace insurance. Let us hope that our experiences in this war will confirm all our fellow citizens in the wisdom and urgency of such a course.

Further than that let me suggest that armament research and development for victory now and peace in the years to come can best be carried forward by the scientific, technical and industrial establishments of our country. Armament progress needs the impetus of free competitive research. It will never be at its best under bureaucratic control, but will thrive only when the brain power of our nation at large is given to the task. Thus I would urge that armament research, after the peace is won, be carried on for the Ordnance Department of the Army by such institutions as The Franklin Institute. Here we should come with our problems in peace as we are doing in war. Here we should lay before you the strategic and tactical needs of our fighting forces. Here we should delegate to you the responsibility for which you are so preeminently equipped. Here we should tell you simply that our soldiers need a weapon of certain characteristics and that certain researches in the field of pure and applied science are required if our needs are to be met. Here, and in countless other research laboratories similar to those of The Franklin Institute, in our universities, laboratories and other private institutions, this work should be done. The traditional viewpoint of Army Ordnance is to utilize the facilities of private industry. This applies to the laboratory as well as to the factory.

At the moment our great concern is victory. Once the victory is won our interest in weapons' preparedness should be as keen as it is at this moment. We should be prepared to embark upon a program of Ordnance research and development, and you of Franklin Institute should continue on the work you are now doing. The future of our military preparedness in the United States is both a question of manpower and weapon power. The weapon power will be attained not by vast accumulations of guns and armaments in storehouses, but in the laboratories such as yours here at Franklin Institute. Our universities will train not only incipient soldiers but future scientists, who, while they chart the peaceful progress of our nation, will not be unmindful of our weapon needs. American science and industry have wrought great things for victory in this war. Theirs is the responsibility to continue our armament research and development hand in hand in

the years of peace. Their cooperative endeavor will be an assurance of peace. If war comes again, in this relationship will be a guarantee that by maintaining our weapons' superiority we will again save countless lives and treasure by our scientific readiness. On Ordnance research and development, with the help of The Franklin Institute, and other technical and scientific laboratories, rests an important element of our structure for peace in the days to come.

Southern Longleaf Pine Disease Fought With Fire.—Sanitation by fire has been found by forest pathologists of the U. S. Department of Agriculture to be the only economically feasible means of controlling the brown spot needle blight of southern longleaf pine. Extreme care must be taken, however, in the use of fire in the woods, and fire sanitation measures should be undertaken only under the direction of experts.

This fungus disease defoliates the seedlings and control of it is of primary importance in perpetuating the longleaf forest type in an area approximating 9,000,000 acres.

Technical Bulletin No. 870, "The Brown Spot Needle Blight of Pine Seedlings," describes how a single ground fire in winter, in badly diseased areas, destroys a large part of the infective material. This permits retention of the needles for two growing seasons, which apparently furnishes the stimulus for vigorous height growth.

The author of the bulletin, Paul V. Siggers of the Bureau of Plant Industry, Soils, and Agricultural Engineering, suggests that where there is much infective material, the burning over of pine areas to get rid of brown spot will have to be done on a large scale. Even an area of 1,000 acres may not be large enough for this purpose when surrounded by an extensive source of the disease. Such burning, however, should be done only when it is certain that adequate control is available. The facts brought out in the study, when combined with proper methods of controlled burning, are expected to provide the basis for regenerating the longleaf pine forests.

R. H. O

THE EXTENSION OF THE HEAVISIDE EXPANSION THEOREM TO THE EQUATIONS OF ENGINEERING AND PHYSICS IN CURVILINEAR ORTHOGONAL COORDINATES.

BY

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SYNOPSIS.

Ever since Heaviside's Expansion Theorem and his operational methods have been available for the solution of transient circuit problems, attempts have been made to extend similar operational methods to the solution of field problems. These methods have not as yet been developed to such a degree as to be employed in field problems that require partial differential equations with more than two independent variables. Such problems are those involving the propagation of electromagnetic or elastic waves, the conduction of heat through solids, the propagation of acoustic waves in air and the distribution of flux in the airgap of electrical machinery all of which may involve one, two, or three dimensions.

This paper undertakes to solve such problems by operational methods similar to Heaviside's, in those types of coordinates of the greatest usefulness to engineers, namely curvilinear orthogonal coordinates. The result is given in formulae which are broader than the Expansion Theorem, being applicable to one, two, or three dimensions and include the Expansion Theorem as a special case.

Four formulae are derived for the field due to a point source in a given space for the following cases:

(1) LaPlace-Poisson equation for steady state conditions.
(2) The equation of conduction of heat.
(3) Maxwell's wave equation.
(4) The wave equation for dissipative medium.

These results are very similar and have many elements in common. It is further shown that by restricting the type of problem to be solved to a certain form (the Green's Function) basic solutions of one, two, or three dimensional equations of the type specified can be expressed in the form of separate functions for each of the variables. These separate functions can be listed in tables similar to tables of integrals and will have a corresponding value in eliminating repetition in mathematical operations.

From the solution given in such tables (for the Green's Function) the solution for other problems can be derived by well known theorems without the necessity of solving the differential equation again for these different conditions.

1. INTRODUCTION.

In problems in Engineering and Physics where the distribution of a field in space is to be determined, the equation to be solved is of the type

$$\nabla^2 v = g_2 \frac{\partial^2 v}{\partial t^2} + g_1 \frac{\partial v}{\partial t} + g_0 \rho, \qquad (1.1)$$

where in curvilinear orthogonal coordinates [1]

[1] See for instance Carslaw, "Conduction of Heat," 2nd Ed., 1921, p. 11, or Byerly, "Fourier Series," p. 239, in which a different notation is used.

$$\nabla^2 = \frac{1}{\lambda\mu\nu}\left[\frac{\partial}{\partial\xi}\left(\frac{\mu\nu}{\lambda}\frac{\partial}{\partial\xi}\right) + \frac{\partial}{\partial\eta}\left(\frac{\nu\lambda}{\mu}\frac{\partial}{\partial\eta}\right) + \frac{\partial}{\partial\zeta}\left(\frac{\lambda\mu}{\nu}\frac{\partial}{\partial\zeta}\right)\right]. \qquad (1.2)$$

When g_1 and g_2 equal zero the equation corresponds to Poisson's equation. When g_2 is equal to zero, it becomes the equation of conduction of heat. When g_1 is equal to zero it becomes Maxwell's wave equation and in the form as given in (1.1) it may be considered as the wave equation for a dissipative medium. The equation is also applicable to problems in sound, hydrodynamics, elasticity, etc.

In this paper an expression for the solution of equation (1.1) is derived for a particular type of conditions which may be referred to as those for the generalized Green's Function for the space under consideration. The problem may be stated as follows:

A point source (of heat, electricity, force, etc.) which may be instantaneous or permanent is applied at the point ξ_1, η_1, ζ_1 at the time t_1 (being there continuously for a permanent point source). At the walls of the enclosure the conditions to be satisfied are of the form

$$hv + \partial v/\partial n = 0, \qquad (1.3)$$

or as an alternative are such that v and $\partial v/\partial n$ are the same respectively at each end of the enclosure. Find the resulting distribution of temperature, electric force, vibration, etc.

The expression in curvilinear orthogonal coordinates for the solution of (1.1) subject to these conditions might be considered as a generalization of the Heaviside Expansion Theorem. For any particular type of coordinates such as rectangular, cylindrical, spherical, etc., we can substitute in it the appropriate associated functions such as the sine and cosine, Bessel Functions, or Legendre functions and obtain the required solution. Thus it is not necessary to return to the differential equation and solve it again each time the coordinate system is changed.

It will be noted that the solution is limited to the field due to a point source in a given space with suitable boundary conditions, so that it corresponds to the Green's Function for the space. This is not as much of a limitation as might appear at first sight since theorems are available which allow other types of problems to be solved when the Green's Function is known. For example, Maxwell [2] gives a theorem by which the potential inside a space with given potentials on the boundary can be calculated when the appropriate Green's Function is known. Carslaw [3] gives another theorem due to Minnigerode by which the temperature in a body, due to a volume distribution of heat or due to given temperatures over the surface of a body can be found when the Green's Function is known. Other theorems of this nature are available.

[2] "Electricity and Magnetism," 3rd Ed., 1904, Vol. 1, p. 133.

[3] "Math. Theory of the Conduction of Heat," 2nd Ed., 1921, p. 171.

Similar devices may be used in other cases. For instance, when it is desired to determine the field due to a moving point source, the procedure to follow is to find the field due to an instantaneous point source and then by integration find the field due to the moving point source. Hence, when the Green's Function for a given space is known, solutions for other distributions may be found by the use of such theorems and it is not necessary to return to the differential equation to solve it for another set of conditions.

By thus standardizing the conditions under which the differential equation is solved, we gain several advantages. It permits us to concentrate upon the solution of one type of problem to which the method used in this paper is peculiarly adapted and avoids the introduction of conditions in the problem which make the application of this method difficult. It also makes possible the limiting of the number of solutions of the differential equation to comparatively few cases so that these can be put in a set of tables which will be available for future use. Such a set of tables has been prepared by the author, but limitations of space prevent further discussion of this here.

In an earlier paper [4] the solution for the field due to a point charge was worked out for specific cases and a suggestion was made as to how the general solution might be derived. In the present paper this work is carried out and a solution in terms of curvilinear orthogonal coordinates derived explicitly.

The method followed in this paper belongs to the class usually referred to as the "operational method." Knowing the result obtained in this manner, it was possible to verify it by the "classical method" which it is hoped to show in a later paper. This is mentioned so that those who may not have confidence in the "operational method" may know that the "classical method" gives the same result. As the "operational method" was chronologically the first to give the solution and is so straightforward in its application, it was thought desirable to use it in this paper.

2. PRINCIPAL FEATURES OF THE OPERATIONAL SOLUTION.

In this method the solution is derived directly from the partial differential equations and the required boundary conditions by methods similar to those used by Heaviside in the development of his expansion theorem. This involves the expansion of differential operators of integral degree into partial fractions of the form

$$\frac{1}{f(D)} F = \sum_{a_n} \frac{1}{(D - a_n)\left[\dfrac{df(D)}{dD}\right]_{D=a_n}} F, \tag{2.1}$$

[4] J. J. Smith, JOURNAL FRANKLIN INSTITUTE, 200, p. 804.

where D is a differential operator with respect to any variable, F denotes a function of that variable, and $a_0, a_1, \cdots$ are the zeros of $f(D)$ regarded as a function of D.

The method also involves the solution of a differential equation representing conditions in a circuit or a space where an exciting agent either suddenly applied or there continuously, such as a force, a charge of electricity or heat, etc., produces the resulting condition which it is required to determine. In the usual mathematical method such equations are generally solved by setting up the general solution with the arbitrary constants. The value of these constants is then obtained by certain terminal conditions which may not always be easy to determine from the physical problem. In the present work the differential equation is written in a form which gives the result directly. The method has been given by the author previously but in order to emphasize it a simple example previously discussed in detail will be briefly given here.[5]

An electrical circuit containing an inductance and resistance is initially in a state of rest. At the time $t = t_0$ an E.M.F. of constant value E is suddenly applied, required the resulting current.

The differential equations to be solved are

$$L\frac{di}{dt} + Ri = 0 \qquad \text{when} \qquad t < t_0, \tag{2.2}$$

$$L\frac{di}{dt} + Ri = E \qquad \text{when} \qquad t > t_0. \tag{2.3}$$

These two equations can be written in the form

$$L\frac{di}{dt} + Ri = EH(t, t_0) \tag{2.4}$$

by use of the Heaviside unit function $H(t, t_0)$ defined as follows

$$\begin{aligned} H(t, t_0) &= 0 \quad \text{when} \quad t < t_0, \\ H(t, t_0) &= 1 \quad \text{when} \quad t > t_0, \\ 0 \leqslant H(t, t_0) &\leqslant 1 \quad \text{when} \quad t = t_0. \end{aligned} \tag{2.5}$$

Since the right-hand side of (2.4) is equal to 0 or E depending on whether $t < t_0$ or $t > t_0$, we write two solutions for i. As it is given that the circuit is initially in a state of rest we have

$$i = 0 \qquad \text{when} \qquad t < t_0. \tag{2.6}$$

We also have, using the general solution,

$$i = C_1 + C_2 e^{-R(t-t_0)/L} \qquad \text{when} \qquad t > t_0. \tag{2.7}$$

[5] JOURNAL FRANKLIN INSTITUTE, **200**, 1925, p. 669, et seq.

These two solutions can be written in the single form

$$i = (C_1 + C_2 e^{-R(t-t_0)/L})H(t, t_0), \tag{2.8}$$

where the term $H(t, t_0)$ on the right ensures that value of i is zero when $t < t_0$. Substitute (2.8) in (2.4) recognizing that $H(t, t_0)$ is a function of t and obtain

$$C_1 RH(t, t_0) + (C_1 + C_2 e^{-R(t-t_0)/L})dH(t, t_0)/dt = EH(t, t_0). \tag{2.9}$$

Now when $t \neq t_0$, $dH(t, t_0)/dt = 0$. Hence we must have

$$C_1 RH(t, t_0) = EH(t, t_0), \tag{2.10}$$

and using this relation in (2.9) we have

$$(C_1 + C_2 e^{-R(t-t_0)/L})dH(t, t_0)/dt = 0. \tag{2.11}$$

But the value of $dH(t, t_0)/dt$ is not zero at $t = t_0$ since for this point $dH(t, t_0) = 1$. Hence the other term must be zero and thus

$$(C_1 + C_2 e^{-R(t-t_0)/L})_{t=t_0} = 0, \tag{2.12}$$

whence

$$C_1 = -C_2. \tag{2.13}$$

But from (2.10)

$$C_1 = E/R. \tag{2.14}$$

Thus the final solution on substituting in (2.8) is

$$i = E(1 - e^{-R(t-t_0)/L})H(t, t_0)/R. \tag{2.15}$$

The extension of this procedure to other problems such as the derivation of the Expansion Theorem will be found in the papers previously referred to. It is unnecessary to go into further detail here as the applications to be made in the following will not require any operations which are essentially different from the above.

When these two methods are used it will be found that the ensuing determination of the Green's Function from the partial differential equations of the Laplace-Poisson type for steady state conditions and the equations of heat conduction or Maxwell's wave equations for transient conditions reduces itself practically to a matter of algebra, and the difficulties encountered in the usual method in the determination of the unknown constants disappear.

In a general problem such as the present, the solution of a particular problem first would undoubtedly be helpful in enabling the quantities involved to appear more realistic in the sense that they would involve definite known functions rather than general functions which are assumed to be solutions of some general differential equation. A number

of specific examples have been worked out previously [6] and these may be compared with the more general solutions given here.

3. LAPLACE-POISSON EQUATION IN CURVILINEAR ORTHOGONAL COORDINATES.

The following problem will now be solved. It is required to find the potential due to a permanent point charge at the point ξ_1, η_1, ζ_1 inside a given space assuming that generalized conditions of the type $\partial v/\partial n + hv = 0$ hold at the boundaries. This will be called the steady state Green's Function for this space.

The position of any point in the space is given in curvilinear orthogonal coordinates by the intersection of the three orthogonal coordinates ξ = a constant (3.1), η = a constant (3.2), and ζ = a constant (3.3).

The boundaries of the space within which it is desired to find the potential are given to be $\xi = a$ (3.4), $\xi = b$ (3.5), $\eta = c$ (3.6), $\eta = d$ (3.7), $\zeta = e$ (3.8) and $\zeta = f$ (3.9). It will be assumed that these boundaries are all at finite distances and also that they are not at singular points, as unless this restriction is made fractional differential coefficients may enter in the work and introduce complications. The solution for these special cases can readily be found from the results given here by finding the solution for boundaries at finite distances or at non-singular points and then taking the limit of the solution thus obtained as the boundaries approach infinity or the singular points.

The equations to be solved are as follows: [7]

$$\frac{1}{\lambda\mu\nu}\left[\frac{\partial}{\partial\xi}\left(\frac{\mu\nu}{\lambda}\frac{\partial v}{\partial\xi}\right)+\frac{\partial}{\partial\eta}\left(\frac{\nu\lambda}{\mu}\frac{\partial v}{\partial\eta}\right)+\frac{\partial}{\partial\zeta}\left(\frac{\lambda\mu}{\nu}\frac{\partial v}{\partial\zeta}\right)\right]=g_0\rho, \qquad (3.10)$$

$$\left.\begin{aligned} -\frac{1}{\lambda}\frac{\partial v}{\partial\xi}+h_1v=0 \quad &\text{when} \quad \xi=a \ (3.11)\\ \frac{1}{\lambda}\frac{\partial v}{\partial\xi}+h_2v=0 \quad &\text{when} \quad \xi=b \ (3.12)\end{aligned}\right\} (b>a)$$

$$\left.\begin{aligned} -\frac{1}{\mu}\frac{\partial v}{\partial\eta}+h_3v=0 \quad &\text{when} \quad \eta=c \ (3.13)\\ \frac{1}{\mu}\frac{\partial v}{\partial\eta}+h_4v=0 \quad &\text{when} \quad \eta=d \ (3.14)\end{aligned}\right\} (d>c)$$

$$\left.\begin{aligned} -\frac{1}{\nu}\frac{\partial v}{\partial\zeta}+h_5v=0 \quad &\text{when} \quad \zeta=e \ (3.15)\\ \frac{1}{\nu}\frac{\partial v}{\partial\zeta}+h_6v=0 \quad &\text{when} \quad \zeta=f \ (3.16)\end{aligned}\right\} (f>e)$$

[6] J. J. Smith, JOURNAL FRANKLIN INSTITUTE, 200, 1925, pp. 791–804.

[7] Byerly, "Fourier Series," p. 239, with a change in notation for the form of Poisson's equation.

$$\lim_{\xi=\xi_1+0} v = \lim_{\xi=\xi_1-0} v \quad (3.17)$$

$$\lim_{\eta=\eta_1+0} v = \lim_{\eta=\eta_1-0} v \quad (3.18)$$

$$\lim_{\zeta=\zeta_1+0} v = \lim_{\zeta=\zeta_1-0} v \quad (3.19)$$

and

$$\rho = 0 \quad \text{when} \quad \xi \neq \xi_1, \quad \eta \neq \eta_1, \quad \text{and} \quad \zeta \neq \zeta_1, \quad (3.20)$$

$$\int_{\zeta_1-0}^{\zeta_1+0}\int_{\eta_1-0}^{\eta_1+0}\int_{\xi_1-0}^{\xi_1+0} \rho\lambda\mu\nu d\xi d\eta d\zeta = Q. \quad (3.21)$$

4. SOLUTION OF EQUATIONS WITH RESPECT TO ξ.

Write

$$\alpha^2 = \frac{1}{\lambda\mu\nu}\left[\frac{\partial}{\partial\eta}\left(\frac{\nu\lambda}{\mu}\frac{\partial}{\partial\eta}\right) + \frac{\partial}{\partial\zeta}\left(\frac{\lambda\mu}{\nu}\frac{\partial}{\partial\zeta}\right)\right], \quad (4.1)$$

where α^2 may be a function [8] of ξ. Then equation (3.10) can be written

$$\frac{1}{\lambda\mu\nu}\frac{\partial}{\partial\xi}\left(\frac{\mu\nu}{\lambda}\frac{\partial v}{\partial\xi}\right) + \alpha^2 v = g_0\rho. \quad (4.2)$$

Since $\rho = 0$ when $\xi \neq \xi_1$ we may write

$$\frac{1}{\lambda\mu\nu}\frac{\partial}{\partial\xi}\left(\frac{\mu\nu}{\lambda}\frac{\partial v}{\partial\xi}\right) + \alpha^2 v = 0 \quad \text{when} \quad \xi \neq \xi_1. \quad (4.3)$$

This is a second order differential equation in ξ and thus has two independent solutions. Let them be $F_1(\alpha, \xi)$ and $F_2(\alpha, \xi)$. It should be understood in this abbreviated notation that α may represent a number of subscripts of a particular function. For instance, if $F_1(\alpha, \xi)$ represents $J_n(b, x)$ in a particular case then α determines both the n and the b.

These two independent solutions may be used to write two equations for v,

$$v_1 = F_1(\alpha, \xi)C_{11} + F_2(\alpha, \xi)C_{12}, \qquad a \leqslant \xi \leqslant \xi_1, \quad (4.4)$$

$$v_2 = F_1(\alpha, \xi)C_{13} + F_2(\alpha, \xi)C_{14}, \qquad \xi_1 \leqslant \xi \leqslant b, \quad (4.5)$$

where C_{11}, C_{12}, C_{13}, and C_{14} are functions of η and ζ but are constants with respect to ξ. They are written after the functions $F_1(\alpha, \xi)$ and $F_2(\alpha, \xi)$ since α from (4.1) is a differential operator.

The reason for writing the two equations in (4.4) and (4.5) is that since ρ is not zero at $\xi = \xi_1$ it is not permissible to assume that either (4.4) or (4.5) alone would satisfy the equation (4.2) at this point. In fact they do not.

[8] A more usual notation would be $\alpha^2(\xi)$ to indicate it may be a function of ξ. This, however, would make the resulting equations unnecessarily complicated in writing.

Three of the four constants C_{11}, C_{12}, C_{13}, and C_{14} can be determined by the use of equations (3.11), (3.12), and (3.17). To determine the fourth constant another equation is required which will now be derived from the differential equation (4.2).

From equations (4.4) and (4.5) the following equation can be written which is valid over the whole range.

$$v = H(\xi_1, \xi)v_1 + H(\xi, \xi_1)v_2, \qquad a \leqslant \xi \leqslant b, \tag{4.6}$$

where $H(\xi, \xi_1)$ is as previously defined in (2.5) and evidently from that equation

$$H(\xi_1, \xi) = 1 - H(\xi, \xi_1) \tag{4.7}$$

and

$$dH(\xi_1, \xi)/d\xi = -dH(\xi, \xi_1)/d\xi. \tag{4.8}$$

It is seen from these definitions of $H(\xi, \xi_1)$ and $H(\xi_1, \xi)$ that (4.6) corresponds to (4.4) when $\xi < \xi_1$ and it corresponds to (4.5) when $\xi > \xi_1$.

To find the value of the fourth constant referred to above the value of v given by (4.6) is substituted in the differential equation (4.2) in the following manner. Performing the differentiation step by step we have

$$\frac{\mu\nu}{\lambda}\frac{\partial v}{\partial \xi} = \frac{\mu\nu}{\lambda}\left[H(\xi_1, \xi)\frac{\partial v_1}{\partial \xi} + H(\xi, \xi_1)\frac{\partial v_2}{\partial \xi} + (v_2 - v_1)\frac{dH(\xi, \xi_1)}{\partial \xi}\right]. \tag{4.9}$$

The last term follows from (4.8). When $\xi \neq \xi_1$, we have $dH(\xi, \xi_1)/d\xi = 0$ and thus

$$\frac{\mu\nu}{\lambda}\frac{\partial v}{\partial \xi} = \frac{\mu\nu}{\lambda}\left[H(\xi_1, \xi)\frac{\partial v_1}{\partial \xi} + H(\xi, \xi_1)\frac{\partial v_2}{\partial \xi}\right]. \tag{4.10}$$

This [9] is also true for $\xi = \xi_1$ since for this point $v_2 \to v_1$ from (3.17).

[9] This was not done in J. F. I., 200, 1925, p. 793, and an equation corresponding to (4.9) was carried through instead of (4.10). This resulted in the introduction of a factor 2 in the result which was dropped for reasons explained there. This point probably merits further study and discussion. Several methods of approach present themselves. One might be as follows. It can be shown that if we assume the potential in the immediate neighborhood of a point source Q is Q/r where r is the distance from the point to the charge then the third term on the right hand side of (4.9) vanishes compared with the other two thus leading to (4.10). For example, we may write

$$v = \{Q/\lambda|d(\xi - \xi_1)|\} + f(\xi - \xi_1),$$

where $f(\xi)$ contains no singularities in the space under consideration. Hence writing the last term in (4.9) in differential form we have

$$[(v_2)_{\xi_1+\epsilon} - (v_1)_{\xi_1-\epsilon}]dH(\xi, \xi_1) = [f(\epsilon) - f(-\epsilon)]dH(\xi, \xi_1),$$

which is therefore finite as $\epsilon \to 0$. The other terms in the square bracket in (4.9) written in differential form are

$$H(\xi_1, \xi)(\partial v_1/\lambda\partial\xi)\lambda d\xi \qquad \text{and} \qquad H(\xi, \xi_1)(\partial v_2/\lambda\partial\xi)\lambda d\xi.$$

They tend to the value $(Q/[\lambda d(\xi - \xi_1)]^2)\lambda d\xi$ or $Q/\lambda d(\xi - \xi_1)$ which tends to infinity as $\xi \to \xi_1$. Hence, $(v_2)_{\xi_1+\epsilon} - (v_1)_{\xi_1-\epsilon}$ vanishes compared with these terms as $\xi \to \xi_1$.

Differentiating again

$$\frac{1}{\lambda\mu\nu}\frac{\partial}{\partial\xi}\left(\frac{\mu\nu}{\lambda}\frac{\partial v}{\partial\xi}\right)=\frac{1}{\lambda\mu\nu}\left[H(\xi_1,\xi)\frac{\partial}{\partial\xi}\left(\frac{\mu\nu}{\lambda}\frac{\partial v_1}{\partial\xi}\right)+H(\xi,\xi_1)\frac{\partial}{\partial\xi}\left(\frac{\mu\nu}{\lambda}\frac{\partial v_2}{\partial\xi}\right)\right]+\frac{1}{\lambda^2}\left(-\frac{\partial v_1}{\partial\xi}+\frac{\partial v_2}{\partial\xi}\right)\frac{dH(\xi,\xi_1)}{d\xi}. \quad (4.11)$$

Insert this in (4.2) and obtain

$$\frac{H(\xi_1,\xi)}{\lambda\mu\nu}\left[\frac{\partial}{\partial\xi}\left(\frac{\mu\nu}{\lambda}\frac{\partial v_1}{\partial\xi}\right)+\alpha^2 v_1\right]+\frac{H(\xi,\xi_1)}{\lambda\mu\nu}\left[\frac{\partial}{\partial\xi}\left(\frac{\mu\nu}{\lambda}\frac{\partial v_2}{\partial\xi}\right)+\alpha^2 v_2\right]+\frac{1}{\lambda^2}\left(-\frac{\partial v_1}{\partial\xi}+\frac{\partial v_2}{\partial\xi}\right)\frac{dH(\xi,\xi_1)}{d\xi}=g_0\rho. \quad (4.12)$$

But from (4.3) the coefficients of $H(\xi_1,\xi)$ and $H(\xi,\xi_1)$ are zero and thus (4.12) reduces to [10]

$$\frac{1}{\lambda}\left(-\frac{\partial v_1}{\partial\xi}+\frac{\partial v_2}{\partial\xi}\right)\frac{dH(\xi,\xi_1)}{d\xi}=g_0\rho\lambda. \quad (4.13)$$

Now $dH(\xi,\xi_1)/d\xi$ is zero at all points except when $\xi=\xi_1$ and at this point $dH(\xi,\xi_1)=1$. Also assume

$$\frac{1}{\lambda}\text{ is finite when }\xi=\xi_1. \quad (4.14)$$

It will be finite in practical problems.[11] We can thus integrate (4.13) from ξ_1-0 to ξ_1+0 and obtain

$$\frac{1}{\lambda}\left(-\frac{\partial v_1}{\partial\xi}+\frac{\partial v_2}{\partial\xi}\right)=\int_{\xi_1-0}^{\xi_1+0}g_0\rho\lambda\,d\xi \quad \text{when} \quad \xi=\xi_1. \quad (4.15)$$

This is the fourth equation required to determine the four constants C_{11}, C_{12}, C_{13}, and C_{14} in equations (4.4) and (4.5). These equations can now be written down explicitly as follows: [12]
From (3.11) and (4.4)

$$[F_1'(\alpha,a)-h_1F_1(\alpha,a)]C_{11}+[F_2'(\alpha,a)-h_1F_2(\alpha,a)]C_{12}=0. \quad (4.16)$$

From (3.12) and (4.5)

$$[F_1'(\alpha,a)+h_2F_1(\alpha,b)]C_{13}+[F_2'(\alpha,b)+h_2F_2(\alpha,b)]C_{14}=0. \quad (4.17)$$

[10] The analogy between this equation and that commonly used in one-dimensional problems $-dv_1/dn+dv_2/dn=4\pi\sigma$ will be noted.

[11] It may not be in special cases. In such cases the simplest method is to find the solution corresponding to a point ξ_2 at which $1/\lambda$ is finite and then take the limit of this solution, if it exists, as $\xi_2\to\xi_1$.

[12] In this paper $F_1'(\alpha,a)$ denotes the value of $dF_1(\alpha,\xi)/\lambda d\xi$ when $\xi=a$.

From (3.17), (4.4), and (4.5)

$$F_1(\alpha, \xi_1)C_{11} + F_2(\alpha, \xi_1)C_{12} - F_1(\alpha, \xi_1)C_{13} - F_2(\alpha, \xi_1)C_{14} = 0, \quad (4.18)$$

and from (4.4), (4.5), and (4.15)

$$-F_1'(\alpha, \xi_1)C_{11} - F_2'(\alpha, \xi_1)C_{12} + F_1'(\alpha, \xi_1)C_{13} + F_2'(\alpha, \xi_1)C_{14} = \int_{\xi_1-0}^{\xi_1+0} g_0\rho\lambda d\xi, \qquad \text{when} \qquad \xi = \xi_1. \quad (4.19)$$

These four equations can be solved for the four constants C_{11}, C_{12}, C_{13}, and C_{14}. As their values are not needed explicitly they will not be written down. Substitute their values in (4.4), (4.5), and (4.6) and the following expression for v is obtained:

$$v = \Xi^* \int_{\xi_1-0}^{\xi_1+0} g_0\rho\lambda d\xi, \quad (4.20)$$

where

$$\Xi^* = \frac{A_1(\alpha, \xi)A_2(\alpha, \xi_1)H(\xi_1, \xi) + A_2(\alpha, \xi)A_1(\alpha, \xi_1)H(\xi, \xi_1)}{K_1(\alpha, \xi_1)D_1(\alpha)} \quad (4.21)$$

and

$$A_1(\alpha, \xi) = [F_2'(\alpha, a) - h_1F_2(\alpha, a)]F_1(\alpha, \xi) - [F_1'(\alpha, a) - h_1F_1(\alpha, a)]F_2(\alpha, \xi), \quad (4.22)$$

$$A_2(\alpha, \xi) = [F_2'(\alpha, b) + h_2F_2(\alpha, b)]F_1(\alpha, \xi) - [F_1'(\alpha, b) + h_2F_1(\alpha, b)]F_2(\alpha, \xi), \quad (4.23)$$

$$D_1(\alpha) = [F_1'(\alpha, a) - h_1F_1(\alpha, a)][F_2'(\alpha, b) + h_2F_2(\alpha, b)] - [F_2'(\alpha, a) - h_1F_2(\alpha, a)][F_1'(\alpha, b) + h_2F_1(\alpha, b)], \quad (4.24)$$

$$K_1(\alpha, \xi_1) = F_2(\alpha, \xi_1)F_1'(\alpha, \xi_1) - F_1(\alpha, \xi_1)F_2'(\alpha, \xi_1). \quad (4.25)$$

We may readily verify this solution by substitution in (3.11), (3.12), (3.17), and (4.15). It can be shown that the value of $K_1(\alpha, \xi_1)$ can be written in a simple form. This is a general property of the solutions of this type of second order differential equation.[13] The result is readily obtained by substituting $F_1(\alpha, \xi)$ and $F_2(\alpha, \xi)$ successively in (4.3). Multiplying the first equation thus obtained by $F_2(\alpha, \xi)$ and the second by $F_1(\alpha, \xi)$ and subtracting the following result is obtained.

$$F_2(\alpha, \xi)\frac{\partial}{\partial \xi}\left(\frac{\mu\nu}{\lambda}\frac{\partial F_1(\alpha, \xi)}{\partial \xi}\right) - F_1(\alpha, \xi)\frac{\partial}{\partial \xi}\left(\frac{\mu\nu}{\lambda}\frac{\partial F_2(\alpha, \xi)}{\partial \xi}\right) = 0. \quad (4.26)$$

[13] See for instance Gray, Mathews, and McRobert, "Bessel Functions," 2nd Ed., 1922, p. 24, for the relation for Bessel Functions.

On integrating this with respect to ξ and then changing ξ to ξ_1 we obtain

$$\left(\frac{\mu\nu}{\lambda}\right)_{\xi=\xi_1}[F_2(\alpha, \xi_1)F_1'(\alpha, \xi_1) - F_1(\alpha, \xi_1)F_2'(\alpha, \xi_1)] = \text{constant}, \quad (4.27)$$

and thus

$$K(\alpha, \xi_1)(\mu\nu/\lambda)_{\xi=\xi_1} = \text{constant}.$$

The result in eq. (4.20) completes the solution of the differential equations in (3.10) or (4.2) with respect to the variable ξ. But since from (4.1) α represents differential operators with respect to η and ζ, the result is still in the form of a differential equation with respect to η which can be solved in a manner very similar to the method just used with respect to ξ.

5. SOLUTION OF EQUATIONS WITH RESPECT TO η.

Expand the expression for v in (4.20) in partial fractions, the summation being over the roots of the denominator, and obtain

$$v = \Xi \frac{1}{\alpha^2 - \alpha_p^2}\int_{\xi_1-0}^{\xi_1+0} g_0\rho\lambda d\xi, \quad (5.1)$$

where [14]

$$\Xi = \sum_{\alpha_p}{}' \frac{2\alpha_p[F_2'(\alpha_p, b) + h_2F_2(\alpha_p, b)]}{F_2'(\alpha_p, a) - h_1F_2(\alpha_p, a)} \frac{A_1(\alpha_p, \xi)A_1(\alpha_p, \xi_1)}{\left[\frac{d}{d\alpha}K_1(\alpha, \xi_1)D_1(\alpha)\right]_{\alpha=\alpha_p}}, \quad (5.2)$$

and [15] $\alpha_0, \alpha_1, \alpha_2, \cdots$ are the positive zeros of $K_1(\alpha, \xi_1)D_1(\alpha)$. The first term Ξ on the right-hand side of (5.1) is fully known since α_p is a real number, and the other symbols are defined in the equations following (4.21).

It will be noted that in (5.1) there is no difference in the expression for v when $\xi < \xi_1$ and $\xi > \xi_1$ so that there is only one expression for v over the range $a \leqslant \xi \leqslant b$. This follows from the fact that when $D_1(\alpha) = 0$, $A_1(\alpha, \xi)$ is proportional to $A_2(\alpha, \xi)$ and the coefficients of the terms $H(\xi_1, \xi)$ and $H(\xi, \xi_1)$ in (4.20) give rise to identical expressions.

It is now necessary to evaluate the coefficient of Ξ in (5.1) which

[14] The $'$ on the summation sign indicates that one-half the value of the expression is taken for α_0.

[15] Since the roots generally occur in pairs, i.e. if α_n is a root $-\alpha_n$ is also a root, the expansion has actually been taken over the roots α^2. In cases where this is not so, such as for the Legendre Functions, the necessary modification can readily be made. In some cases, such as for the Legendre Functions, the pairing may occur in the form $\alpha_n + \frac{1}{2}$ and $-(\alpha_n + \frac{1}{2})$ and the necessary modification to the above can readily be made. Some such form of pairing occurs in all the cases studied to date. If pairing did not occur, then from the definition of α^2 in (4.1) the value of α would be a fractional differential operator and a consideration of these is outside the scope of this paper.

can be written

$$G_{1p} = \frac{1}{\alpha^2 - \alpha_p^2} \int_{\xi_1-0}^{\xi_1+0} g_0 \rho \lambda d\xi. \tag{5.3}$$

Substitute the value of α^2 from (4.1) and rearrange, obtaining

$$\frac{1}{\lambda\mu\nu}\left[\frac{\partial}{\partial\eta}\left(\frac{\nu\lambda}{\mu}\frac{\partial G_{1p}}{\partial\eta}\right) + \frac{\partial}{\partial\zeta}\left(\frac{\lambda\mu}{\nu}\frac{\partial G_{1p}}{\partial\zeta}\right)\right] - \alpha_p^2 G_{1p} = \int_{\xi_1-0}^{\xi_1+0} g_0 \rho \lambda d\xi. \tag{5.4}$$

This equation is very similar to equation (3.10) except that the differential operation with respect to ξ has been replaced by the number α_p, the term $g_0\rho$ has been replaced by the integral on the right-hand side, and the dependent variable is now G_{1p} instead of v. As a result of this similarity practically identical methods can now be used to solve the equation with respect to η as were already used to obtain a solution with respect to ξ. Write

$$\beta^2 = \frac{1}{\lambda\mu\nu}\frac{\partial}{\partial\zeta}\left(\frac{\lambda\mu}{\nu}\frac{\partial}{\partial\zeta}\right) - \alpha_p^2. \tag{5.5}$$

Equation (5.4) becomes

$$\frac{1}{\lambda\mu\nu}\frac{\partial}{\partial\eta}\left(\frac{\nu\lambda}{\mu}\frac{\partial G_{1p}}{\partial\eta}\right) + \beta^2 G_{1p} = \int_{\xi_1-0}^{\xi_1+0} g_0 \rho \lambda d\xi, \tag{5.6}$$

and when $\eta \neq \eta_1$ the right-hand side of this equation is zero. Thus it may be treated in a manner similar to (4.2). If $F_3(\beta, \eta)$ and $F_4(\beta, \eta)$ are independent solutions of this equation the solution can be written

$$G_{1p} = H(\eta_1, \eta)G_{1p1} + H(\eta, \eta_1)G_{1p2}, \tag{5.7}$$

where

$$G_{1p1} = F_3(\beta, \eta)C_{21} + F_4(\beta, \eta)C_{22}, \qquad c \leqslant \eta \leqslant \eta_1, \tag{5.8}$$

$$G_{1p2} = F_3(\beta, \eta)C_{23} + F_4(\beta, \eta)C_{24}, \qquad \eta_1 \leqslant \eta \leqslant d, \tag{5.9}$$

and C_{21}, C_{22}, C_{23}, and C_{24} may be functions of ζ but are constant with respect to η.

An inspection of (5.1) shows it may be written

$$v = \sum_p{}' f(\xi)G_{1p}, \tag{5.10}$$

where $f(\xi)$ is the term after the summation sign in the expression for Ξ in (5.2). Hence if the G_{1p} terms satisfy the same terminal conditions with respect to η as are given for v, the sum of the G_{1p} terms will also satisfy these conditions and thus the value of v given by (5.1) will also. Thus the terminal conditions for v with respect to η are now rewritten

as terminal conditions for G_{1p} with respect to η from (3.13), (3.14), and (3.18) respectively as follows:

$$-\frac{1}{\mu}\frac{\partial G_{1p}}{\partial \eta} + h_3 G_{1p} = 0 \qquad \text{when} \qquad \eta = c, \tag{5.11}$$

$$\frac{1}{\mu}\frac{\partial G_{1p}}{\partial \eta} + h_4 G_{1p} = 0 \qquad \text{when} \qquad \eta = d, \tag{5.12}$$

and

$$\operatorname*{Limit}_{\eta=\eta_1+0} G_{1p} = \operatorname*{Limit}_{\eta=\eta_1-0} G_{1p}. \tag{5.13}$$

A fourth equation, obtained by substituting (5.7) in (5.6) in a manner identical with that already used in deriving equation (4.15), is

$$\frac{1}{\mu}\left(-\frac{\partial G_{1p1}}{\partial \eta} + \frac{\partial G_{1p2}}{\partial \eta}\right) = \int_{\eta_1-0}^{\eta_1+0}\int_{\xi_1-0}^{\xi_1+0} g_0\rho\lambda\mu d\xi d\eta \text{ when } \eta = \eta_1. \tag{5.14}$$

From (5.11), (5.12), (5.13), and (5.14) four equations similar to (4.16) to (4.19) can be obtained and solved for the constants C_{21}, C_{22}, C_{23}, and C_{24}. Since they are not needed explicitly only the final result obtained by substituting these values in (5.7), (5.8), and (5.9) will be given. This can readily be written down by comparison with (4.20) and is as follows:

$$v = \Xi H^* \int_{\eta_1-0}^{\eta_1+0}\int_{\xi_1-0}^{\xi_1+0} g_0\rho\lambda\mu d\xi d\eta, \tag{5.15}$$

where

$$H^* = \frac{A_3(\beta,\eta)A_4(\beta,\eta_1)H(\eta_1,\eta) + A_4(\beta,\eta)A_3(\beta,\eta_1)H(\eta,\eta_1)}{K_2(\beta,\eta_1)D_2(\beta)}, \tag{5.16}$$

and

$$A_3(\beta,\eta) = [F_4'(\beta,c) - h_3F_4(\beta,c)]F_3(\beta,\eta) - [F_3'(\beta,c) - h_3F_3(\beta,c)]F_4(\beta,\eta), \tag{5.17}$$

$$A_4(\beta,\eta) = [F_4'(\beta,d) + h_4F_4(\beta,d)]F_3(\beta,\eta) - [F_3'(\beta,d) + h_4F_3(\beta,d)]F_4(\beta,\eta), \tag{5.18}$$

$$D_2(\beta) = [F_3'(\beta,c) - h_3F_3(\beta,c)][F_4'(\beta,d) + h_4F_4(\beta,d)] - [F_4'(\beta,c) - h_3F_4(\beta,c)][F_3'(\beta,d) + h_4F_3(\beta,d)], \tag{5.19}$$

$$K_2(\beta,\eta_1) = F_4(\beta,\eta_1)F_3'(\beta,\eta_1) - F_3(\beta,\eta_1)F_4'(\beta,\eta_1). \tag{5.20}$$

Equation (5.15) completes the solution of the differential equation (3.10) or (5.4) with respect to η. But since from (5.5) β represents a differential operator with respect to ζ, the result is still in the form of a

differential equation. The next step is to consider (5.15) as a differential equation with respect to ζ which can be solved in a manner identical with that just used to evaluate (4.20) with respect to η.

6. SOLUTION OF EQUATIONS WITH RESPECT TO ζ.

Expand the term H^* in the expression for v as given by (5.15) in partial fractions, the summation being over the roots of the denominator, and obtain

$$v = \Xi H \frac{1}{\beta^2 - \beta_q^2} \int_{\eta_1-0}^{\eta_1+0} \int_{\xi_1-0}^{\xi_1+0} g_0 \rho \lambda \mu d\xi d\eta, \tag{6.1}$$

where

$$H = \sum_{\beta_q}{}' \frac{2\beta_q[F_4'(\beta_q, d) + h_4 F_4(\beta_q, d)]}{F_4'(\beta_q, d) - h_3 F_4(\beta_q, d)} \frac{A_3(\beta_q, \eta) A_3(\beta_q, \eta_1)}{\left[\frac{d}{d\beta} K_2(\beta, \eta_1) D_2(\beta)\right]_{\beta=\beta_q}}, \tag{6.2}$$

and $\beta_0, \beta_1, \beta_2, \cdots$ are the positive zeros of $K_2(\beta, \eta)D_2(\beta)$. It is again assumed that if β_q is a positive zero, $-\beta_q$ is also a zero. The first part ΞH on the right-hand side of (6.1) is fully known since β_q is a real number, and the other symbols are defined in the equations following (5.16).

It will be noted that in (6.1) there is no difference in the expression for v when $\eta < \eta_1$ and $\eta > \eta_1$ so that there is only one expression for v over the range $c \leqslant \eta \leqslant d$. This follows since when $D_2(\beta) = 0$, $A_3(\beta, \eta)$ is proportional to $A_4(\beta, \eta)$.

It is now necessary to evaluate the coefficient of ΞH in (6.1) which can be written

$$G_{2q} = \frac{1}{\beta^2 - \beta_q^2} \int_{\eta_1-0}^{\eta_1+0} \int_{\xi_1-0}^{\xi_1+0} g_0 \rho \lambda \mu d\xi d\eta. \tag{6.3}$$

This expression is similar to equation (5.3) and its solution is obtained by the same method used for that equation. Write

$$\gamma_r^2 = -\alpha_p^2 - \beta_q^2. \tag{6.4}$$

Then (6.3) becomes, using the value of β^2 in (5.5),

$$\frac{1}{\lambda\mu\nu} \frac{\partial}{\partial\zeta} \left(\frac{\lambda\mu}{\nu} \frac{\partial G_{2q}}{\partial\zeta} \right) + \gamma_r^2 G_{2q} = \int_{\eta_1-0}^{\eta_1+0} \int_{\xi_1-0}^{\xi_1+0} g_0 \rho \lambda \mu d\xi d\eta. \tag{6.5}$$

This is similar to (5.6).

The solution of this equation may be written

$$G_{2q} = H(\zeta_1, \zeta) G_{2q1} + H(\zeta, \zeta_1) G_{2q2}, \tag{6.6}$$

where

$$G_{2q1} = F_5(\gamma, \zeta) C_{31} + F_6(\gamma, \zeta) C_{32}, \qquad e \leqslant \zeta \leqslant \zeta_1, \tag{6.7}$$

$$G_{2q2} = F_5(\gamma, \zeta) C_{33} + F_6(\gamma, \zeta) C_{34}, \qquad \zeta_1 \leqslant \zeta \leqslant f. \tag{6.8}$$

The boundary conditions with respect to ζ in (3.15), (3.16), and (3.19) are applied to G_{2q} for the same reasons that the boundary conditions with respect to η were applied to G_{1p} as in (5.11), (5.12), and (5.13) and thus

$$-\frac{1}{\nu}\frac{\partial G_{2q}}{\partial \zeta} + h_5 G_{2q} = 0, \qquad \zeta = e, \tag{6.9}$$

$$\frac{1}{\nu}\frac{\partial G_{2q}}{\partial \zeta} + h_4 G_{2q} = 0, \qquad \zeta = f, \tag{6.10}$$

and

$$\lim_{\zeta=\zeta_1+0} G_{2q} = \lim_{\zeta=\zeta_1-0} G_{2q}. \tag{6.11}$$

A fourth equation is obtained in the manner used previously by substituting (6.6) in (6.5) which gives

$$\frac{1}{\nu}\left(-\frac{\partial G_{2q1}}{\partial \zeta} + \frac{\partial G_{2q2}}{\partial \zeta}\right) = \int_{\zeta_1-0}^{\zeta_1+0}\int_{\eta_1-0}^{\eta_1+0}\int_{\xi_1-0}^{\xi_1+0} g_0\rho\lambda\mu\nu d\xi d\eta d\zeta \qquad \text{when} \qquad \zeta = \zeta_1. \tag{6.12}$$

From this equation and the three preceding ones the values of C_{31}, C_{32}, C_{33}, and C_{34} can be obtained. However, they are not needed explicitly and we shall give only the final result obtained by substituting these values in (6.6), (6.7), and (6.8) as follows:

$$v = \Xi H Z^* \int_{\zeta_1-0}^{\zeta_1+0}\int_{\eta_1-0}^{\eta_1+0}\int_{\xi_1-0}^{\xi_1+0} g_0\rho\lambda\mu\nu d\xi d\eta d\zeta, \tag{6.13}$$

where

$$Z^* = \frac{A_5(\gamma,\zeta)A_6(\gamma,\zeta_1)H(\zeta_1,\zeta) + A_6(\gamma,\zeta)A_5(\gamma,\zeta_1)H(\zeta,\zeta_1)}{K_3(\gamma,\zeta_1)D_3(\gamma)}, \tag{6.14}$$

and

$$A_5(\gamma,\zeta) = [F_6'(\gamma,e) - h_5F_6(\gamma,e)]F_5(\gamma,\zeta) - [F_5'(\gamma,e) - h_5F_5(\gamma,e)]F_6(\gamma,\zeta), \tag{6.15}$$

$$A_6(\gamma,\zeta) = [F_6'(\gamma,f) + h_6F_6(\gamma,f)]F_5(\gamma,\zeta) - [F_5'(\gamma,f) + h_6F_5(\gamma,f)]F_6(\gamma,\zeta), \tag{6.16}$$

$$D_3(\gamma) = [F_5'(\gamma,e) - h_5F_5(\gamma,e)][F_6'(\gamma,f) + h_6F_6(\gamma,f)] - [F_6'(\gamma,e) - h_5F_6(\gamma,e)][F_5'(\gamma,f) + h_6F_5(\gamma,f)], \tag{6.17}$$

$$K_3(\gamma,\zeta_1) = F_6(\gamma,\zeta_1)F_5'(\gamma,\zeta_1) - F_5(\gamma,\zeta_1)F_6'(\gamma,\zeta_1). \tag{6.18}$$

Using (3.21) it is seen that (6.13) can be written

$$v = g_0 Q \Xi H Z^*. \tag{6.19}$$

This is then the desired solution for a permanent point source inside the space with boundaries given by (3.4) to (3.9) where the equation of the medium is given by (3.10) and the conditions at the boundaries by (3.11) to (3.19). This may also be called the Steady State Green's Function for this space, subject to these boundary conditions.

In obtaining this result the solution was first derived with respect to ξ, then with respect to η, and finally with respect to ζ. This resulted in a different form for Z^* from the form for Ξ and H. For instance, in the case of Ξ a term Ξ^* given by (4.21) was first found and from it Ξ in (5.2) was derived by an operation involving expansion in partial fractions. Similarly we note the relation between H^* and H. But in the case of the variable ζ with respect to which the equation was last solved there is in (6.13) no term corresponding to Z.

If, however, the order is changed and the equation solved with respect to the variable ξ first, ζ second, and then η, the following solution is obtained.

$$v = g_0 Q \Xi Z H^*, \tag{6.26}$$

where

$$Z = \sum_{\gamma_r}{}' \frac{2\gamma_r[F_6'(\gamma_r, f) + h_6 F_6(\gamma_r, f)]}{F_6'(\gamma_r, e) - h_5(\gamma_r, e)} \frac{A_5(\gamma_r, \zeta) A_5(\gamma_r, \zeta_1)}{\left[\frac{d}{d\gamma} K_3(\gamma, \zeta_1) D_3(\gamma)\right]_{\gamma=\gamma_r}}, \tag{6.21}$$

which may be written down directly on comparison with (5.2) or (6.2), and $\gamma_0, \gamma_1, \gamma_2, \cdots$ are the positive zeros of $K_3(\gamma, \zeta_1) D_3(\gamma)$. It is again assumed that if γ_r is a positive zero, $-\gamma_r$ is also a zero. The symbols in this expression are defined in the equations following (6.14).

Similarly if the equation is solved with respect to η first, ζ second, and then ξ it is found that

$$v = g_0 Q H Z \Xi^*. \tag{6.22}$$

The latter solution is known as the ξ form, that in (6.20) as the η form and that in (6.19) as the ζ form. Examples of these three forms in the solution of specific problems will be found in Gray, Matthews, and MacRobert, and in Dougall.[16]

A fourth form which is more symmetrical than these forms can be obtained by expanding the "starred term" in these solutions in partial fractions. The result obtained from each of them is the same and is

$$v = g_0 Q \Xi H Z, \tag{7.26}$$

where $\alpha_p^2 + \beta_q^2 + \gamma_r^2 = 0$. Since this solution is in the form of a triple summation it is not as generally used as the ξ, η and ζ forms which only involve a double summation.

[16] "Bessel Functions," 2nd Ed., 1922, p. 103.

In these equations Ξ is given by (5.2), Ξ^* by (4.21), H by (6.2), H^* by (5.16), Z by (6.21), and Z^* by (6.14).

7. ALTERNATIVE FORM OF BOUNDARY CONDITIONS.

In some problems the boundary condition with respect to one or more of the coordinates may not be of the form $\partial v/\partial n_i + hv = 0$. For instance, with cylindrical coordinates r, z and ϕ if the space extends from $\phi = -\pi$ to $\phi = \pi$ with no specific boundaries the conditions to be met are

$$[v]_{\phi=-\pi} = [v]_{\phi=\pi}, \tag{7.1}$$

and

$$[\partial v/\partial \phi]_{\phi=-\pi} = [\partial v/\partial \phi]_{\phi=\pi}. \tag{7.2}$$

Assume therefore that such a condition exists for the ξ coordinate so that the boundary conditions instead of being given by (3.11) and (3.12) are

$$[v]_{\xi=a} = [v]_{\xi=b}, \tag{7.3}$$

$$[\partial v/\partial \xi]_{\xi=a} = [\partial v/\partial \xi]_{\xi=b}, \tag{7.4}$$

and the remaining equations (3.13) to (3.21) remain the same. Then proceed without change in the section entitled "Solution of equations with respect to ξ" as far as equations (4.4), (4.5), and (4.6) which may be written

$$v_1 = F_1(\alpha, \xi)C_{41} + F_2(\alpha, \xi)C_{42}, \qquad a \leqslant \xi \leqslant \xi_1, \tag{7.5}$$

$$v_2 = F_1(\alpha, \xi)C_{43} + F_2(\alpha, \xi)C_{44}, \qquad \xi_1 \leqslant \xi \leqslant b, \tag{7.6}$$

and

$$v = H(\xi_1, \xi)v_1 + H(\xi, \xi_1)v_2. \tag{7.7}$$

Three of the constants, C_{41}, C_{42}, etc., can then be determined as in that section by using equations (7.3), (7.4), and (3.17). The fourth equation is derived by substituting the value of v as given by (7.7) in the original differential equation (3.10) written in the form (4.1) and obtaining an equation similar to (4.15) as follows:

$$\frac{1}{\lambda}\left(-\frac{\partial v_1}{\partial \xi} + \frac{\partial v_2}{\partial \xi}\right) = \int_{\xi_1-0}^{\xi_1+0} g_0\rho\lambda d\xi \qquad \text{when} \qquad \xi = \xi_1. \tag{7.8}$$

The resulting equations are respectively from (7.5), (7.6), (3.17), and (7.8).

$$F_1(\alpha, a)C_{41} + F_2(\alpha, a)C_{42} - F_1(\alpha, b)C_{43} - F_2(\alpha, b)C_{44} = 0, \tag{7.9}$$

$$F_1'(\alpha, a)C_{41} + F_2'(\alpha, a)C_{42} - F_1'(\alpha, b)C_{43} - F_2'(\alpha, b)C_{44} = 0, \tag{7.10}$$

$$F_1(\alpha, \xi_1)C_{41} + F_2(\alpha, \xi_1)C_{42} - F_1(\alpha, \xi_1)C_{43} - F_2(\alpha, \xi_1)C_{44} = 0, \tag{7.11}$$

$$- F_1'(\alpha, \xi_1)C_{41} - F_2'(\alpha, \xi_1)C_{42} + F_1'(\alpha, \xi_1)C_{43} + F_2'(\alpha, \xi_1)C_{44}$$

$$= \int_{\xi_1-0}^{\xi_1+0} g_0\rho\lambda d\xi \qquad \text{when} \qquad \xi = \xi_1. \quad (7.12)$$

These four equations can be solved for the four constants C_{41}, C_{42}, C_{43}, and C_{44}. As their values are not needed explicitly they will not be written down. Substitute their values in (7.5), (7.6), and (7.7) and the following expression for v is obtained.

$$v = \Xi'^* \int_{\xi_1-0}^{\xi_1+0} g_0\rho\lambda d\xi, \quad (7.13)$$

where

$$\Xi'^* = \frac{[A_7(\alpha, \xi_1)F_1(\alpha, \xi) + A_8(\alpha, \xi_1)F_2(\alpha, \xi)]H(\xi_1, \xi)}{K_4(\alpha, \xi_1)D_4(\alpha)} + \frac{[A_9(\alpha, \xi_1)F_1(\alpha, \xi) + A_{10}(\alpha, \xi_1)F_2(\alpha, \xi)]H(\xi, \xi_1)}{K_4(\alpha, \xi_1)D_4(\alpha)}, \quad (7.14)$$

and

$$A_7(\alpha, \xi) = F_1(\alpha, \xi)[F_2(\alpha, a)F_2'(\alpha, b) - F_2'(\alpha, a)F_2(\alpha, b)] + F_2(\alpha, \xi)\{F_1(\alpha, b)[F_2'(\alpha, a) - F_2'(\alpha, b)] - F_1'(\alpha, b)[F_2(\alpha, a) - F_2(\alpha, b)]\}, \quad (7.15)$$

$$A_8(\alpha, \xi) = F_2(\alpha, \xi)[F_1(\alpha, a)F_1'(\alpha, b) - F_1'(\alpha, a)F_1(\alpha, b)] + F_1(\alpha, \xi)\{F_2(\alpha, b)[F_1'(\alpha, a) - F_1'(\alpha, b)] - F_2'(\alpha, b)[F_1(\alpha, a) - F_1(\alpha, b)]\}, \quad (7.16)$$

$$A_9(\alpha, \xi) = F_1(\alpha, \xi)[F_2(\alpha, a)F_2'(\alpha, b) - F_2'(\alpha, a)F_2(\alpha, b)] + F_2(\alpha, \xi)\{F_1(\alpha, a)[F_2'(\alpha, a) - F_2'(\alpha, b)] - F_1'(\alpha, a)[F_2(\alpha, a) - F_2(\alpha, b)]\}, \quad (7.17)$$

$$A_{10}(\alpha, \xi) = F_2(\alpha, \xi)[F_1(\alpha, a)F_1'(\alpha, b) - F_1'(\alpha, a)F_1(\alpha, b)] + F_1(\alpha, \xi)\{F_2(\alpha, a)[F_1'(\alpha, a) - F_1'(\alpha, b)] - F_2'(\alpha, a)[F_1(\alpha, a) - F_1(\alpha, b)]\}, \quad (7.18)$$

$$D_4(\alpha) = [F_1(\alpha, a) - F_1(\alpha, b)][F_2'(\alpha, a) - F_2'(\alpha, b)] - [F_1'(\alpha, a) - F_1'(\alpha, b)][F_2(\alpha, a) - F_2(\alpha, b)], \quad (7.19)$$

and

$$K_4(\alpha, \xi_1) = F_2(\alpha, \xi_1)F_1'(\alpha, \xi_1) - F_1(\alpha, \xi_1)F_2'(\alpha, \xi_1). \quad (7.20)$$

Equation (7.13) replaces (4.20) in the previous section. This completes the solution with respect to ξ. Expand (7.13) in partial fractions with respect to the roots of the denominator as in the previous section entitled "Solution of Equations with Respect to η" in expanding equa-

tion (4.20) in the form given in (5.1). Then

$$v = \Xi' \frac{1}{\alpha^2 - \alpha_p^2} \int_{\xi_1 - 0}^{\xi_1 + 0} g_0 \rho \lambda d\xi, \tag{7.21}$$

where

$$\Xi' = \sum_{\alpha_p}' \frac{2\alpha_p [A_7(\alpha, \xi_1) F_1(\alpha, \xi) + A_8(\alpha, \xi_1) F_2(\alpha, \xi)]}{\left[\frac{d}{d\alpha} K_4(\alpha, \xi_1) D_4(\alpha) \right]_{\alpha = \alpha_p}}, \tag{7.22}$$

and α_0, α_1, α_2, $\cdots$ are positive zeros of $K_4(\alpha, \xi) D_4(\alpha)$. It is again assumed that if α_p is a zero, $-\alpha_p$ is also a zero. The symbols in this expression are defined in the equations following (7.14).

From this point the work proceeds as in the previous sections giving the final solution corresponding to (6.19)

$$v = g_0 Q \Xi' H Z^*, \tag{7.23}$$

which is the ζ form of solution. Similarly the η form is found to be

$$v = g_0 Q \Xi' Z H^*. \tag{7.24}$$

To find the ξ form, solve first with respect to η and ζ, and then it will be found that

$$v = g_0 Q Z H \Xi'^*, \tag{7.25}$$

where Ξ'^* is given by (7.14).

Equations (7.23), (7.24), or (7.25) give the solution for a permanent point source inside the space with boundaries given by (3.4) to (3.9) where the equation of the medium is given by (3.10) and the conditions at the boundaries are given by (7.3), (7.4), and (3.13) to (3.19). Ξ' is given by (7.22), Ξ'^* by (7.14), H by (6.2), H^* by (5.16), Z by (6.21) and Z^* by (6.14).

8. SOLUTION OF EQUATION OF HEAT CONDUCTION.

The previous results have given, for two different types of boundary conditions, the solution of (1.1) or (3.10) which corresponds to the steady state condition. When there is variation with respect to time the Laplace-Poisson equation (3.10) is replaced in the case of problems in heat conduction by [17]

$$\frac{1}{\lambda \mu \nu} \left[\frac{\partial}{\partial \xi} \left(\frac{\mu \nu}{\lambda} \frac{\partial v}{\partial \xi} \right) + \frac{\partial}{\partial \eta} \left(\frac{\nu \lambda}{\mu} \frac{\partial v}{\partial \eta} \right) + \frac{\partial}{\partial \zeta} \left(\frac{\lambda \mu}{\nu} \frac{\partial v}{\partial \zeta} \right) \right] = g_1 \frac{\partial v}{\partial l} + g_0 \rho_1. \tag{8.1}$$

[17] In the usual form of the equation given in textbooks the term ρ_1 is omitted. This term is essential in the solution given in the following. A derivation of the equation including it will be found in J. F. I. 200, 1925, p. 789.

The boundary conditions with respect to the space coordinates ξ, η and ζ will be taken to be the same as in (3.11) to (3.19).

The solution will be restricted to that corresponding to an instantaneous point source of heat[18] at the point ξ_1, η_1, ζ_1 at the time t_1. For this case ρ_1 is given by

$$\rho_1 = 0 \quad \text{when} \quad \xi \neq \xi_1, \quad \eta \neq \eta_1, \quad \zeta \neq \zeta_1, \quad \text{and} \quad t \neq t_1, \tag{8.2}$$

and

$$\int_{t_1-0}^{t_1+0}\int_{\zeta_1-0}^{\zeta_1+0}\int_{\eta_1-0}^{\eta_1+0}\int_{\xi_1-0}^{\xi_1+0} \rho_1 \lambda\mu\nu d\xi d\eta d\zeta dt = Q_1. \tag{8.3}$$

A boundary condition is needed with respect to t. For the present case of an instantaneous point source at $t = t_1$ this will be taken to be that the medium is in a state of rest prior to the time t_1, so that

$$v = 0 \quad \text{when} \quad t < t_1. \tag{8.4}$$

To solve these equations proceed as before. In the section entitled "Solution of equation with respect to ξ" replace (4.1) by

$$\alpha^2 = \frac{1}{\lambda\mu\nu}\left[\frac{\partial}{\partial\eta}\left(\frac{\nu\lambda}{\mu}\frac{\partial}{\partial\eta}\right) + \frac{\partial}{\partial\zeta}\left(\frac{\lambda\mu}{\nu}\frac{\partial}{\partial\zeta}\right)\right] - g_1\frac{\partial}{\partial t}. \tag{8.5}$$

It will be found that the addition of the operator with respect to time does not affect the solution with respect to ξ and the same value of v is obtained as in (4.20) where α is now given by (8.5) instead of (4.1). Proceeding further in the section entitled "Solution of Equations with respect to η" equation (5.5) is replaced by

$$\beta^2 = \frac{1}{\lambda\mu\nu}\frac{\partial}{\partial\zeta}\left(\frac{\lambda\mu}{\nu}\frac{\partial}{\partial\zeta}\right) - g_1\frac{\partial}{\partial t} - \alpha_p^2, \tag{8.6}$$

and the same value of v is obtained as in (5.15) where β is now given by (8.6) instead of (5.5). To evaluate further equation (6.4) is replaced by

$$\gamma^2 = -g_1\frac{\partial}{\partial t} - \alpha_p^2 - \beta_q^2, \tag{8.7}$$

and then the work follows closely the section "Solution of Equations with respect to ζ" down to (6.13).

In this expression on account of the value of γ^2 given by (8.7) the denominator of Z^* now contains a differential operator with respect to time and hence the result is still in the form of a differential equation.

[18] For further discussion see Byerly, "Fourier Series," p. 93, or Carslaw, "The Conduction of Heat," 2nd. Ed., 1921, p. 149.

By expanding in partial fractions with respect to γ the following is obtained

$$v = \Xi HZ \frac{1}{\gamma^2 - \gamma_r^2} \int_{f_1-0}^{f_1+0} \int_{\eta_1-0}^{\eta_1+0} \int_{\xi_1-0}^{\xi_1+0} g_0 \rho_1 \lambda \mu \nu d\xi d\eta d\zeta. \tag{8.8}$$

To evaluate this write

$$G_{3p} = \frac{1}{\gamma^2 - \gamma_r^2} \int_{f_1-0}^{f_1+0} \int_{\eta_1-0}^{\eta_1+0} \int_{\xi_1-0}^{\xi_1+0} g_0 \rho_1 \lambda \mu \nu d\xi d\eta d\zeta. \tag{8.9}$$

This expression is of the same general form as those previously encountered and its solution is obtained by similar methods. Write for brevity

$$(\alpha_p^2 + \beta_q^2 + \gamma_r^2)/g_1 = g_3. \tag{8.10}$$

Then (8.9) becomes using (8.7)

$$\frac{\partial G_{3p}}{\partial t} + g_3 G_{3p} = \frac{1}{g_1} \int_{f_1-0}^{f_1+0} \int_{\eta_1-0}^{\eta_1+0} \int_{\xi_1-0}^{\xi_1+0} g_0 \rho_1 \lambda \mu \nu d\xi d\eta d\zeta. \tag{8.11}$$

Since $\rho_1 = 0$ when $t \neq t_1$ the right side of the equation is zero when $t \neq t_1$ and thus two solutions must be written, the first being given by applying the boundary condition for v in (8.4) to G_{3p} for the same reason that the boundary conditions with respect to η were applied to G_{1p} in equations (5.11), (5.12), and (5.13). Thus

$$G_{3p1} = 0 \qquad \text{when} \qquad t < t_1, \tag{8.12}$$

and

$$G_{3p2} = C_{51} e^{-g_3(t-t_1)} \qquad \text{when} \qquad t > t_1. \tag{8.13}$$

These may be put in the single form

$$G_{3p} = C_{51} e^{-g_3(t-t_1)} H(t, t_1). \tag{8.14}$$

Substituting this in equation (8.11) it is found that

$$C_{51} \frac{dH(t, t_1)}{dt} = \frac{1}{g_1} \int_{f_1-0}^{f_1+0} \int_{\eta_1-0}^{\eta_1+0} \int_{\xi_1-0}^{\xi_1+0} g_0 \rho_1 \lambda \mu \nu d\xi d\eta d\zeta. \tag{8.15}$$

Integrate both sides of this equation with respect to t from $t_1 - 0$ to $t_1 + 0$ and use (8.3) to obtain

$$C_{51} = g_0 Q_1 / g_1, \tag{8.16}$$

and thus

$$G_{3p} = g_0 Q_1 e^{-g_3(t-t_1)} H(t, t_1)/g_1 = g_0 Q_1 T_1^*. \tag{8.17}$$

On taking the value of g_3 given by (8.10) and writing

$$T_1^* = e^{-[(\alpha_p^2 + \beta_q^2 + \gamma_r^2)(t-t_1)/g_1]} H(t, t_1)/g_1. \tag{8.18}$$

Introducing this value of G_{3p} in (8.8) the final expression for v is obtained,

$$v = g_0 Q_1 \Xi H Z T_1^*. \tag{8.19}$$

This is the expression for the temperature due to an instantaneous point source placed at the point ξ_1, η_1, ζ_1 at the time t_1 in a space which has conditions at the boundaries given by (3.11) to (3.19) and (8.4).

In (8.19) Ξ is given by (5.2), H by (6.2), Z by (6.21) and T_1^* by (8.18). It should be noted in this case that there is only one form of solution which is called the t form. No forms corresponding to the ξ, η, and ζ forms in the steady state solution exist. An asterisk has been placed on the T_1^* to emphasize this.

9. SOLUTION OF THE WAVE EQUATION FOR A DISSIPATIVE MEDIUM.

In the case of the equation of Heat Conduction the differential operator with respect to time is of the first order. In Maxwell's wave equation this operator is of the second order. In the case of wave propagation in a dissipative medium both the first and second order differential operators appear. Although the latter equation is not generally used in textbooks, its solution can be obtained just as directly as the other solutions by the present methods. It will therefore be used in this section and from it in the next section the solution of Maxwell's wave equation will be determined as a limiting condition.

For wave propagation in a dissipative medium, equation (3.10) is replaced by

$$\frac{1}{\lambda\mu\nu}\left[\frac{\partial}{\partial\xi}\left(\frac{\mu\nu}{\lambda}\frac{\partial v}{\partial\xi}\right) + \frac{\partial}{\partial\eta}\left(\frac{\nu\lambda}{\mu}\frac{\partial v}{\partial\eta}\right) + \frac{\partial}{\partial\zeta}\left(\frac{\lambda\mu}{\nu}\frac{\partial v}{\partial\zeta}\right)\right] = g_2\frac{\partial^2 v}{\partial t^2} + g_1\frac{\partial v}{\partial t} + g_0\rho_1. \tag{9.1}$$

The same boundary conditions will be retained with respect to the space coordinates ξ, η, ζ as in the case of the previous solutions which are given in equations (3.11) to (3.19). The solution will also be limited to that corresponding to an instantaneous point source or charge at the point ξ_1, η_1, and ζ_1 at the instant t_1 so that ρ_1 is given by (8.2) and (8.3). The remaining boundary conditions required to complete the solution will be taken to be that the medium was in a state of rest prior to the time t_1 so that equation (8.4) applies.

The work now proceeds as in the case of the equation of Heat Conduction replacing (8.5) by

$$\alpha^2 = \frac{1}{\lambda\mu\nu}\left[\frac{\partial}{\partial\eta}\left(\frac{\nu\lambda}{\mu}\frac{\partial}{\partial\eta}\right) + \frac{\partial}{\partial\zeta}\left(\frac{\lambda\mu}{\nu}\frac{\partial}{\partial\zeta}\right)\right] - g_2\frac{\partial^2}{\partial t^2} - g_1\frac{\partial}{\partial t}, \tag{9.2}$$

and (8.6) by

$$\beta^2 = \frac{1}{\lambda\mu\nu}\frac{\partial}{\partial\zeta}\left(\frac{\lambda\mu}{\nu}\frac{\partial}{\partial\zeta}\right) - \alpha_p{}^2 - g_2\frac{\partial^2}{\partial t^2} - g_1\frac{\partial}{\partial t}, \tag{9.3}$$

and (8.7) by

$$\gamma^2 = -\alpha_p{}^2 - \beta_q{}^2 - g_2\frac{\partial^2}{\partial t^2} - g_1\frac{\partial}{\partial t}, \tag{9.4}$$

finally obtaining an equation identical in form with (8.8).

$$v = \Xi HZ\frac{1}{\gamma^2 - \gamma_r{}^2}\int_{\xi_1-0}^{\xi_1+0}\int_{\eta_1-0}^{\eta_1+0}\int_{\zeta_1-0}^{\zeta_1+0} g_0\rho_1\lambda\mu\nu d\xi d\eta d\zeta, \tag{9.5}$$

where, however, γ^2 is now given by (9.4) instead of (8.7). Write

$$G_{4r} = \frac{1}{\gamma^2 - \gamma_r{}^2}\int_{\xi_1-0}^{\xi_1+0}\int_{\eta_1-0}^{\eta_1+0}\int_{\zeta_1-0}^{\zeta_1+0} g_0\rho_1\lambda\mu\nu d\xi d\eta d\zeta. \tag{9.6}$$

Let

$$g_1/2g_2 = g_4, \tag{9.7}$$

and

$$w^2 + g_4{}^2 = (\alpha_p{}^2 + \beta_q{}^2 + \gamma_r{}^2)/g_2. \tag{9.8}$$

Then from (9.4) and (9.6)

$$\frac{\partial^2 G_{4r}}{\partial t^2} + 2g_4\frac{\partial G_{4r}}{\partial t} + (w^2 + g_4{}^2)G_{4r} = -\frac{1}{g_2}\int_{\xi_1-0}^{\xi_1+0}\int_{\eta_1-0}^{\eta_1+0}\int_{\zeta_1-0}^{\zeta_1+0} g_0\rho_1\lambda\mu\nu d\xi d\eta d\zeta. \tag{9.9}$$

Since from (8.2) $\rho_1 = 0$ when $t \neq t_1$ the right-hand side of this equation is zero when $t \neq t_1$, and thus two solutions must be written, using (8.4) applied to G_{4r} instead of v for the case when $t < t_1$.

$$G_{4r1} = 0 \qquad \text{when} \qquad t < t_1, \tag{9.10}$$

$$G_{4r2} = e^{-g_4(t-t_1)}[C_{61}\sin w(t-t_1) + C_{62}\cos w(t-t_1)] \text{ when } t > t_1. \tag{9.11}$$

These two solutions may be written as one in the form

$$G_{4r} = e^{-g_4(t-t_1)}[C_{61}\sin w(t-t_1) + C_{62}\cos w(t-t_1)]H(t, t_1). \tag{9.12}$$

This value of G_{4r} is now substituted in (9.9) when on using

$$e^{-g_4(t-t_1)}\sin w(t-t_1)\frac{\partial H(t,t_1)}{\partial t} = 0, \tag{9.13}$$

and

$$e^{-g_4(t-t_1)}\cos w(t-t_1)\frac{\partial H(t,t_1)}{\partial t} = \frac{\partial H(t,t_1)}{\partial t}, \tag{9.14}$$

we obtain

$$H(t,t_1)\left[\frac{\partial^2}{\partial t^2}+2g_4\frac{\partial}{\partial t}+(w^2+g_4^2)\right]$$
$$\times\left[C_{61}e^{-g_4(t-t_1)}\sin w(t-t_1)+C_{62}e^{-g_4(t-t_1)}\cos w(t-t_1)\right]$$
$$+\frac{\partial H(t,t_1)}{\partial t}\left[C_{61}w-g_4C_{62}+C_{62}\right]$$
$$+C_{62}\frac{\partial^2 H(t,t_1)}{\partial t}=-\frac{1}{g_2}\int_{\zeta_1-0}^{\zeta_1+0}\int_{\eta_1-0}^{\eta_1+0}\int_{\xi_1-0}^{\xi_1+0}g_0\rho_1\lambda\mu\nu d\xi d\eta d\zeta. \qquad (9.15)$$

The first term on the left side of this equation is zero. Integrate with respect to time from t_1-0 to t_1+0 and obtain

$$\left[C_{61}w-g_4C_{62}+C_{62}\right]+C_{62}\frac{\partial H(t,t_1)}{\partial t}$$
$$=-\frac{1}{g_2}\int_{t_1-0}^{t_1+0}\int_{\zeta_1-0}^{\zeta_1+0}\int_{\eta_1-0}^{\eta_1+0}\int_{\xi_1-0}^{\xi_1+0}g_0\rho_1\lambda\mu\nu d\xi d\eta d\zeta dt$$
$$=-g_0Q_1/g_2 \qquad \text{from} \qquad (8.3). \qquad (9.16)$$

But since $\dfrac{\partial H(t,t_1)}{\partial t}$ is infinite at $t=t_1$, C_{62} must be equal to zero and hence

$$C_{61}=-g_0Q_1/g_2w. \qquad (9.17)$$

Thus from (9.12)

$$G_{4r}=-\frac{g_0Q_1}{g_2w}e^{-g_4(t-t_1)}\sin w(t-t_1)H(t,t_1)=g_0Q_1T_3^*, \qquad (9.18)$$

on taking the values of g_4 and w given by (9.7) and (9.8) and writing

$$T_3^*=-\frac{e^{-g_1(t-t_1)/2g_2}\sin\left[(t-t_1)\right.}{\left.\times\{g_2(\alpha_p^2+\beta_q^2+\gamma_r^2)-g_1^2/4\}^{\frac{1}{2}}/g_2\right]H(t,t_1)}{\left[g_2(\alpha_p^2+\beta_q^2+\gamma_r^2)-g_1^2/4\right]^{\frac{1}{2}}}. \qquad (9.20)$$

Introducing this value of G_{4r} in (9.5), the final expression for v is obtained

$$v=g_0Q_1\Xi HZT_3^* \qquad (9.21)$$

This is the expression for the potential due to an instantaneous point source at the point ξ_1, η_1, ζ_1 at the time t_1 in a dissipative medium for which the differential equation is given by (9.1) and the boundary conditions by (3.11) to (3.19) and (8.4). In (9.21) Ξ is given by (5.2), H by (6.2), Z by (6.21) and T_3^* by (9.20).

10. SOLUTION OF MAXWELL'S WAVE EQUATION.

If g_1 equals zero in (9.1) it becomes Maxwell's wave equation

$$\frac{1}{\lambda\mu\nu}\left[\frac{\partial}{\partial\xi}\left(\frac{\mu\nu}{\lambda}\frac{\partial v}{\partial\xi}\right)+\frac{\partial}{\partial\eta}\left(\frac{\nu\lambda}{\mu}\frac{\partial v}{\partial\eta}\right)+\frac{\partial}{\partial\zeta}\left(\frac{\lambda\mu}{\nu}\frac{\partial v}{\partial\zeta}\right)\right] = g_2\frac{\partial^2 v}{\partial t^2}+g_0\rho_1. \quad (10.1)$$

In the usual derivation of Maxwell's equation given in textbooks ρ_1 denotes a continuous point charge and if the charge is in motion the method of solution used here would not apply since ρ_1 would not be a function of ξ_1, η_1, ζ_1, and t_1 where these are constants. In order that they may be constants even though the charge is in motion so that the present method can be applied it is necessary to assume that ρ_1 is an instantaneous point charge and in this case the solution of (10.1) is obtained from that in the previous section by letting g_1 approach zero. Write

$$T_2^* = \operatorname*{Limit}_{g_1\to 0} T_3^* = -\frac{\sin\left[(t-t_1)(\alpha_p^2+\beta_q^2+\gamma_r^2)^{\frac{1}{2}}/g_2^{\frac{1}{2}}\right]H(t,t_1)}{\left[(\alpha_p^2+\beta_q^2+\gamma_r^2)g_2\right]^{\frac{1}{2}}}. \quad (10.2)$$

Then the equation corresponding to (9.21) becomes

$$v = g_0 Q_1 \Xi H Z T_2^*, \quad (10.3)$$

which is the expression for the potential due to an instantaneous point charge at the point ξ_1, η_1, ζ_1 at the time t_1 in a nondissipative medium to which Maxwell's wave equation given by (10.1) applies subject to the boundary conditions (3.11) to (3.19) and (8.4).

Some care is needed with this concept of an "instantaneous point charge" and in the method of using it. In the case of the equation of heat conduction it was pointed out that there was only one form of solution, namely the t form. This is also the case in the solution of the wave equation if only retarded solutions are considered when the terminal condition with respect to t is given by (8.4). It should be noted, however, that if terminal conditions with respect to t are taken to be

$$-\frac{\partial v}{\partial t}+h_7 v = 0 \quad \text{when} \quad t=\tau_1, \quad (10.4)$$

$$\frac{\partial v}{\partial t}+h_8 v = 0 \quad \text{when} \quad t=\tau_2, \quad (10.5)$$

or alternatively the conditions corresponding to v and $\partial v/\partial t$ having the same values respectively at each end of the interval, solutions of Maxwell's equation can be found in all four forms, namely the ξ, η, ζ, and

t forms. It will be found, however, that these solutions corresponding to (10.4) and (10.5) are a combination of advanced and retarded potentials and are therefore not acceptable on the basis of the present electromagnetic theory. Thus they have not been derived here.

A more detailed discussion of the application of the expression for the field due to an "instantaneous point charge" to the derivation of the fields due to moving point charges will be given in a subsequent paper.

11. DISCUSSION OF RESULTS.

In the solutions which have been derived and are summarized in the following section it will be noted that these contain many common elements. It will also be noted that the solution with respect to any one of the coordinates is independent of the solution with respect to the other coordinates. Thus for instance, the expression for Ξ is unaffected by a change in the boundary conditions with respect to η or ζ. The result of this is that the expressions for Ξ, H, and Z can be evaluated for the various coordinate systems and the results put in the form of tables which will be available for future use like a table of integrals. Such tables have been compiled by the author from the expressions given in this paper for rectangular, cylindrical and spherical coordinates. These tables have been checked satisfactorily with known results where they were available and also contain many new results. Examples of Green's Functions which may be readily derived from the formula in this paper will be found in the references given below.[19] By the use of Duhamel's theorem [20] or other similar theorems [21] which give the solution for other types of conditions when the expression for the field due to a point charge is known, these tables can then be applied to many other types of problems including all types commonly encountered in the usual textbooks.

There are other aspects of these solutions which space does not permit discussing here but are of considerable interest. For instance, it can be shown from these solutions that the expressions Ξ, H, and Z are intimately connected with the Theory of Fourier Series and that, for example, a general expression for a Fourier Series is of the type

$$f(\xi, \eta, \zeta) = \int\int\int f(\xi_1, \eta_1, \zeta_1)\Xi HZ\lambda\mu\nu d\xi_1 d\eta_1 d\zeta_1, \qquad (11.1)$$

where the integration is over appropriate intervals with respect to each

[19] Carslaw, "Theory of Heat Conduction," 2nd Ed., 1921, Chap. X. Gray, Mathews, and McRobert, "Bessel Functions," 2nd Ed., 1922, Chap. IX; Dougall, *Proc. Edin. Math. Soc.*, Vol. XVIII.

[20] Berg, "Heaviside's Operational Calculus," p. 67, or Bush, "Operational Calculus."

[21] See for instance, Maxwell, "Electricity and Magnetism," 3rd Ed., Vol. 1, p. 133; Carslaw, "The Conduction of Heat," 2nd Ed., 1921, p. 169.

coordinate. Equation (11.1) in certain cases can be broken down into separate Fourier Series with respect to each of the coordinates. Having obtained that result by the present method it was then shown that the same expansion could be obtained by the usual Fourier method of assuming a series for the function and determining the values of the coefficients.

With this Fourier expansion established in the usual way it was possible to obtain the solution of the heat equation for an instantaneous initial volume distribution of heat, and from this for an instantaneous point source of heat. The result checked with that given here. From the solution for an instantaneous point source of heat the solution for the steady state condition was obtained by integration and also checked with the above result. As other questions come up in connection with the wave equation in both dissipative and non-dissipative media they will not be discussed here.

12. SUMMARY.

Explicit expressions have been derived for the field due to a point source or charge in a given space in four cases. Since the results are scattered through the paper references to them will be collected here. The meaning of the symbols can be found by referring to the equation of the same number given in the paper.

(1) Solution of Poisson's equation for a steady state continuous point source

$$v = g_0 QHZ\Xi^*, \tag{6.22}$$

or

$$v = g_0 Q\Xi ZH^*, \tag{6.20}$$

or

$$v = g_0 Q\Xi HZ^*, \tag{6.19}$$

or

$$v = g_0 Q\Xi HZ. \tag{6.26}$$

The first three are the ξ, η, and ζ forms and involve a double summation or integral. The fourth which is equivalent to the first three is in the form of a triple summation or integral.

(2) Solution of equation of heat conduction for an instantaneous point source

$$v = g_0 Q\Xi HZT_1^*. \tag{8.19}$$

(3) Solution of Maxwell's equation for an instantaneous point source.

$$v = g_0 Q\Xi HZT_2^*. \tag{10.3}$$

(4) Solution of the wave equation for a dissipative medium for an instantaneous point source

$$v = g_0 Q\Xi HZT_3^*. \tag{9.21}$$

The boundary conditions will be found given with the equations referred to in each case. When the linear boundary conditions which usually apply are replaced by conditions that v and $\partial v/\partial \xi$ are the same respectively at each end of the interval, Ξ or Ξ^* must be replaced by Ξ' or Ξ'^* as given in (7.14) or (7.22) and similarly for the other co-ordinates.

When written together in this manner it is noted that the solutions are very similar to each other and contain many common elements. This permits their compilation in the form of tables previously referred to. It also means that if, for example, we know the solution for a problem in heat conduction involving an instantaneous point source only a slight modification in the factor involving time is necessary to obtain the solution for a steady state continuous point source, or an instantaneous point source in a space in which Maxwell's equation applies or in a space in which the wave equation for a dissipative medium applies. This is evident since if we have the solution corresponding to (8.19) we have only to change T_1^* to 1, T_2^* or T_3^* to get (6.26), (10.3), and (9.21) respectively. It may also be of interest that if in (9.21) we put $H = 1$ and $Z = 1$ we have the Expansion Theorem in the form used by Heaviside for application to the case of a point source on a cable.

The work so far has assumed that the boundaries are at finite distances and are not at singular points. When the boundaries are at infinite distances or approach singular points, difficulties are encountered if an attempt is made to apply the method directly due to the fact that for these special cases the operators may assume forms corresponding to fractional differential coefficients which may not be easy to interpret. However, for these cases the solution may be readily obtained by first solving the equations for the case with finite boundaries and then taking the limit as the boundaries are assumed to approach infinity or a singular point.

NUTRITIONAL ENGINEERING, III: USE OF FOOD AND FOOD-PRODUCTION RESOURCES.

BY

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Reports of the Secretary of Agriculture (1) and of the War Food Administrator (2) emphasize the fact that the American food supply must simultaneously support our armed forces and our civilians, contribute heavily to the needs of our allies, and insofar as possible, help to feed the peoples that allied victories liberate. As Auchter and Hambidge (3) have put it, the demand has been for more of everything; and they describe interestingly how the nation has mobilized both its material and its mental resources. Thus, again in 1943, for the seventh year in succession, the United States exceeded all its previous records in total food production.

The purpose of this paper is to sketch briefly, without technicalities or more than a minimum of statistics, but in as scientifically sound a manner as may be, the nutritional significance of present methods of use of our food-production resources and of the modifications suggested by the current production goals.

The general level of our national food production is about one-third higher than in 1935–39 so that the current allocation of 75 per cent. of the food supply to civilians, of whom there are fewer now, means that our people are on the average more abundantly fed now than then, and with food of better balanced nutritive value. The other allocations were 13 per cent. for our own armed forces, 10 per cent. for lend-lease, and 2 per cent. for special needs such as relief feeding. But this does not mean that the year's production of each kind of food is divided in these proportions, for some foods are obviously more practicable for overseas shipment and stockpiling than others.

Obviously, too, we are facing a period in which there will be an increased need of food to send to the victims of the war before the needs of our own and our allies' armed forces will decrease; and already the reserves of grain with which we entered the war are somewhat depleted.

So while there is no occasion for alarm and no prospect of anything that should be called a "crisis" in our food situation, there is occasion for some mild adjustments as proposed by the Department of Agri-

culture. These will, we believe, be more clearly appreciated when placed in the setting of the following sketch of our use of food and of our national resources for food production. For, as the Secretary has emphasized in his annual report, we are fighting this war without reducing the national level of nutrition. In fact, with present adjustments of production and distribution, and with the increased purchasing power of most low-income families, we are undoubtedly reducing the pre-war prevalence of malnutrition, as have the British with their constructive wartime adjustments of agriculture and food supply.

As Secretary Wickard has written: We want neither scares nor complacency, but information on which to base continued well-directed effort.

The Government encouraged the farmers of the United States to plant about 5 per cent. more acres with emphasis on soybeans and peanuts, other beans and peas, and potatoes. With an increased amount of fat thus produced through soybeans and peanuts, it will not be necessary to spend quite so much grain in carrying the fattening of hogs to such a high degree as in 1942 and 1943. Hogs will come to market at a slightly earlier average age, with less of the extra body fat that experience has shown to be less efficiently produced and utilized; and a somewhat larger fraction of our grain crop will be available for other uses.

In 1942 and the first half of 1943, we could and did expedite our war effort by drawing upon our grain reserves—among other things. Now, with larger numbers of armed forces overseas, we need larger food reserves. For, whereas a three-months' supply is deemed an adequate reserve in providing for civilian trade and for the feeding of troops at home, a nine-months' allowance for troops to be fed abroad should be always either on its way or safely stocked in the overseas supply depots of the Army and Navy. Not only is this important as a direct military precaution; it is also to be remembered that an uncertain but considerable amount of feeding of liberated civilians must be done by the Army —and without any delay—while other agencies are being organized. And increasing numbers of war prisoners and of volunteers from among the liberated peoples may become eligible to receive the generous ration that this country provides for every enlisted man. The effective coordination of the policy represented by current adjustments of agricultural production goals with that of sharing food with other peoples while maintaining good nutrition at home has been emphasized by the National Research Council (4).

Our Federal Bureau of Agricultural Economics uses a three-fold grouping of ways to increase farm production: (*a*) Those which increase the acreage used for agricultural production, (*b*) those which apply improved techniques to produce more products per acre or per animal,

and (*c*) those which shift the type or combination of products grown—for example, a substitution of soybeans for oats, or milk for beef.

Except in a few cases specifically mentioned, increases of production referred to in this paper do not depend upon the assumption of an increase in acreage devoted to agriculture. We are here considering what can be and is being done through a more scientific attitude toward techniques and especially planning shifts.

When science working on the land shows how two blades of grass or bushels of grain can be grown where one grew before, and when, working in the laboratory, it also shows how nutritional knowledge with enough of the right kinds of food can give man a sort of mastery of his life history which he has never realized before, there is opened a large new field of what Franklin liked to call "philosophical and useful knowledge," and which in this Journal we may call *the nutritional engineering of our agricultural resources*.

And in this phrase we say *our* advisedly, for every one of us, as a consumer of food, has a stake in food production and influences it through his "consumer demand."

Obviously the resources of agriculture and of food production are not quite identical throughout, but much the largest part of each is contained in the other, and the parts that do not overlap are interlocked with those that do. Cotton fiber is not a food crop but the seed that grows with it is. We ordinarily distinguish between agriculture and fisheries, but the thoroughness with which the fisheries are "worked" depends largely upon how much meat agriculture is sending to market that season. A year of dustbowls and relative dearth may remind us that "there's never a drought in the sea"; but usually our sea-foods, while welcome as varying our diet, are (as yet *) little more than a relish from the year-round quantitative point of view. So we commit no serious inaccuracy when, having bracketed fish and shellfish with meats in our grouping of foods, we proceed to speak as if our foods all came from agriculture—so nearly is this the case in the quantitative sense.

In terms of their agricultural origin, foods may be first divided between (*a*) *the primary*, or products of *direct food crops;* (*b*) *the secondary*, resulting from the "processing" of crops and pastures through animals; to which might be added (*c*) the *tertiary* or foods still more expensive of resources to produce because they involve the feeding of domestic animal products to animals again as in the case of milk-fed chicken or milk-fed pig.

Usually the foregoing terminology is abbreviated to the simple use of the phrase "direct food crops" in any context in which it is deemed

* Very likely our national fishery resources may sometime be more fully developed.

helpful; while, for further discrimination, increasing use is now made of the classification of food supplies into the following:

Eleven Food Groups: (1) Grain products; (2) dry beans, peas, nuts, oil-seeds; (3) potatoes and sweetpotatoes; (4) green and yellow vegetables; (5) tomatoes and citrus fruits; (6) other fruits and vegetables; (7) milk and its products other than butter; (8) eggs; (9) meats,* including fish and poultry; (10) fats,* including butter and all edible fatty oils; (11) sugars, sirups, and other sweets.

This division of foods into eleven groups, which may then be arranged in any sequence deemed most convenient for the discussion in hand, is proving very practical for use in such varied connections as the agricultural, the economic, the nutritional, and the social conventions of meal-planning.

In the following paragraphs we use the sequence of the eleven food groups above mentioned for the purposes of our discussion of the nature, magnitude, and uses of our major food crops and resources for food production.

Grain products.—The official estimates (5) of the major United States grain crops of 1942 (other than the sorghums) are as follows:

Corn (maize)	3,131,518,000	bushels
Oats	1,349,547,000	"
Wheat	974,176,000	"
Barley	429,167,000	"
Rice	64,549,000	"
Rye	57,673,000	"
Buckwheat	6,636,000	"

If, after taking out seed for the following year, and the grain needed for draft animals, the remainder were moderately milled for human food these grain crops would together yield a food energy value of about 400 million million calories or the equivalent of the entire calorie need of 400 million people or nearly three times our population. Of course, grain products alone do not make an adequate diet. But when we add the fruit and vegetable crops, and the animal products that could be raised on the pastures and roughages which our farms and ranges afford and the byproducts of the moderate milling of the above grains, we should have a much larger total of food calories and perhaps as well balanced nutritionally as are the food supplies of the majority of the world's people. But of course with our present knowledge of the influence of nutrition upon the quality of human life, we would wish any diet of our planning to be nutritionally richer and better balanced than most people's diets *now* are. So let us suppose that an insignificant fraction

* Bacon and salt side are sometimes included with meats; sometimes with fats. Preserves and marmalades are sometimes included with fruits, but usually with sweets.

of land now devoted to growing grain and cotton be diverted to increased production of fruit and vegetables, and that half of the entire grain crop (as well as the milling byproducts of the other half) be fed to farm animals for production of milk, eggs, and meat, in proportions consistent with our best present nutritional knowledge. By such use of its food-production resources the United States would have produced a nutritionally excellent food supply for its own people and as many more. The several statements made by different authorities within the past year, that our farmers could do this if it were necessary, are well within the facts.

The adjustments actually called for are relatively small and simple. Current goals include an increase of about 22 per cent. in the acreage of wheat with little change in the other grain crops. We shall thus be able to send more wheat and flour abroad and also make fuller use of grain products in our own dietaries, while diminishing the intensive grain-feeding of meat animals.

Economy and efficiency in the use of our resources are certainly promoted by the somewhat fuller use of grain products as food. Table 1 shows that in 1942 our people were investing only 11 per cent. of their food money in grain products and receiving therefrom 30 per cent. of the total food calories, 28 per cent. of the food protein, 22 per cent. of the thiamine, and 21 per cent. of the total iron of the dietary (6). Thus on all these points the grain products brought returns of around two to three times their quota; and their contributions of thiamine and iron are relatively larger now that "enrichment" of breadstuffs is more generally practiced.

But while enrichment restores white flour to something like whole-wheat levels of thiamine, iron, and niacin, it does nothing for the proteins which are poorer both in percentage and nutritional efficiency in white than in whole-wheat flour. Whereas whole-wheat protein brings all the nutritionally essential amino acids in fair proportions, white flour protein is so poor in some of these that it definitely needs supplementation by some other protein which contains them more richly—such as the proteins of milk, eggs, meat, or oil-seeds (soybean, peanut, cottonseed).

Wheat is grown in every major region of the United States. It is well to have the expert opinion of the Bureau of Agricultural Economics that our wheat production possibilities are quite flexible and can well be maintained at high levels if desired; for the assurance of ample supplies of bread will be of great value to morale and of real help to the prospect of building a durable peace.

Discriminating studies of summer-fallow needs of different soils and climatic regions, in order to maintain their full productivity for wheat, are now being further developed by the U. S. Department of Agriculture. Industrial demands for wheat—such as to make alcohol to make explo-

sives and synthetic rubber—are also being studied steadily in the endeavor to put our wheat-production resources to best use from the combined viewpoints of the emergency needs of wartime industry and both the present and the permanent place of wheat in the best possible nutrition of ourselves and other peoples.

Rice is produced chiefly (so far as this country is concerned) in Louisiana, Texas, Arkansas, and California, each of which has increased its rice production in recent years. Only moderate further increases could be expected without undue expenditure upon the land and water resources employed.

Buckwheat also admits of only small increases of economic production.

The "feed" grains (corn, oats, barley, and sorghums) plus tame hay occupy more than half the total crop land of the United States. But even so these feed crops would not adequately feed the numbers of livestock we had in this country in 1943. However, there is another large resource for livestock production in the cheap native or wild forage which grows on large areas of range land which is not likely to be brought under cultivation in any future that we can foresee. Far the largest part of this range pasture land is so situated as not to contribute to dairy farming and will doubtless continue to be used almost exclusively for the production of beef cattle and sheep,—with, in some regions, enough horses for picturesque effect and for use in the herding of the cattle.

The sorghums other than those grown for sirup, are usually given an acreage approaching that of barley, or nearly one-fifth that of corn. Much of this crop is fed without having been harvested as grain, but nevertheless makes a large contribution to the grain feeding of livestock.

The best way to expand output of feed in the Northeastern States is to improve the carrying capacity of pastures and the yields of hay. And this is being done, but perhaps not as rapidly as it might be. The methods however are known and are being more generally and effectively taught.

Mature Beans and Peas, Soybeans, and Peanuts.—The production of these crops is being increased in larger percentage than that of the cereal grains. More of the vegetable oils is wanted so that less grain need be expended in the uneconomical production of excessively fat hogs. Here soybeans and peanuts are the crops best suited for expansion. Then, if the policy of slaughtering meat animals at earlier ages and less heavily grain-fed finally brings meat production into equilibrium at a somewhat lowered level, legume protein will presumably find an increased market as an alternative to meat protein. Meanwhile and in any case it is highly prized for the feeding of animals. Recent research has shown that soybean and peanut proteins have high nutritional efficiency both in themselves and as supplementing the proteins

of grain products. The other chief oil-seeds—those of cotton and flax—are expected to remain at about their present acreage and production levels.

While the proteins of the oil-seeds have yet been relatively little used as human food in this country, except in the form of peanuts and peanut butter, they constitute an immense reserve which could be drawn upon if an actual shortage of food-protein should occur. Also our long-familiar forms of beans and peas could well be more frequently used as alternatives for the more expensive high-protein foods. Table 1 shows that the relatively small investment in these foods by American families in 1942 brought from 3-fold to 11-fold returns in food energy

TABLE I.

Relative Cost and Contributions to Nutritive Value of Each Food Group: U.S.D.A., Study of Spending and Saving in Wartime, Spring of 1942, American Family Dietaries.

Food Group.	Per Cent. of Total Food Cost.	Percentage of Total Contributed by Each Food Group.							
		Calories.	Protein.	Calcium.	Iron.	Vitamin A Value.	Thiamine.	Riboflavin.	Vitamin C.
Grain products *	11	30	28	12	21	**	22	9	0
Dry beans, peas, nuts	1	3	5	3	11	**	6	3	0
Potatoes, sweetpotatoes	3	5	3	2	7	6	8	4	13
Green and yellow vegetables	5	1	2	5	8	39	6	5	31
Citrus fruits, tomatoes	6	2	1	3	4	7	6	2	35
Other vegetables and fruits	8	4	2	3	6	6	3	5	13
Milk, cheese, ice cream	17	14	23	65	7	15	8	43	6
Eggs	7	3	8	3	10	7	6	13	0
Meat, poultry, fish	23	10	25	2	21	9	31	15	1
Butter and other fats	10	19	2	**	2	11	4	1	0
Sugars, sirups, sweets	3	8	**	2	3	**	**	**	1
Miscellaneous	2	1	1	**	**	**	**	**	**
Food adjuncts	4	No attempt to estimate nutrients in these							

* 35 per cent. of flour enriched.

** Less than 0.5 per cent.

(calories), protein, calcium, iron, thiamine, and riboflavin. It is hoped that production of these low-cost, high-value foods will be increased as rapidly as the Government recommends, even though we do not know to what extent they will be eaten in this country and to what extent shipped to our allies, who have already taken large amounts.

Full realization of Government goals would increase our crops of ordinary beans and peas together by about one-third of a million acres; of peanuts by about one million; and of soybeans by about three million acres.

The seed oils are interchangeable to such a degree that we may properly think in terms of their total. They can also largely replace lard—are doing so now, and could to a still larger extent if desired. We

need only increase the acreages of soybeans and peanuts without materially decreasing that of cotton or of flax.

Increase of peanut acreage would be made possible largely by use of idle land and by bringing some new land into cultivation. The Upper and Lower Coastal Plains of the Southeast and the Gulf Coast could furnish most of the land. Texas, Arkansas, and Oklahoma also could develop new peanut lands.

*Potatoes and Sweetpotatoes.**—These are bracketed in the customary grouping of foods because they are both high-energy foods and occupy an interchangeable place in meal-planning. They differ in the one respect that potatoes are very poor, and sweetpotatoes rich, in vitamin A value. (The vitamin A *value* is due to the presence of precursors, and not of vitamin A itself.) A glance at Table 1 will show that potatoes and sweetpotatoes taken together bring their full quota or more of food energy, protein, iron, vitamin A value, thiamine, riboflavin, and vitamin C (ascorbic acid). It will, therefore, be to our nutritional and economic advantage to increase our production and consumption of these foods as our government's plan of food production goals recommends.

Production of potatoes and sweetpotatoes can be increased up to any amounts that we can store, transport, and consume. Greater flexibility of dietary habit—eating more of each vegetable one likes, at the time it is most abundantly available—would here be advantageous both to the individual or family and to the general situation. In regions particularly well adapted to potato or sweetpotato growing, it may be kept in mind that in case of an unsaleable surplus beyond storage capacity, it can be fed to hogs as partial substitute for grain, all sound grain being easy to store for long periods. In sufficiently southern regions, the same land may yield sweetpotatoes and some other crop in the same year. The Bureau of Agricultural Economics points out that our consumption is only about two-thirds that of the British who are (1943–44) eating 257 pounds of potatoes per caput per year.

Sweetpotatoes, like peanuts, are regarded by our national Bureau of Agricultural Economics as capable of being advantageously grown in largely increased quantities in some southern areas, by making fuller use of the land. Certainly the sweetpotato field is an excellent illustration of a crop making full use of sunshine and producing high food-energy and vitamin A values. An educational effort might largely increase the consumption of sweetpotatoes in the northern States. We are told, for instance, that many Minnesota families eat sweetpotatoes with gusto on Thanksgiving Day but do not see, or think about, them on other days. Canning may help in meeting the storage problem.

Green and Yellow Vegetables.—The literally green—not merely fresh

* Science writes sweetpotato as one word, as a hint that it does not belong to the potato family.

or immature—and the yellow vegetables * are grouped together because of their relatively high (though variable) vitamin A value. Again this is value, not content: What is actually present is not vitamin A itself but one or more of its precursors, collectively called carotenes or carotenoids. Animals, including man, transform these precursors into vitamin A in their bodies. Of late, there has been some disposition to question whether our species does this quite as efficiently as was supposed and some nutritionists feel that perhaps we should deduct a quarter or a third from the vitamin A values assigned these vegetables. This is still a doubtful point. But if we assume the worst and deduct a third from the vitamin A value of green and yellow vegetables in Table 1 we should still have a 5-fold return of vitamin A value upon the money invested in these vegetables; as well as full- or over-quota returns in calcium, iron, thiamine, riboflavin, and vitamin C. We hope that our national production and consumption of these vegetables will rapidly increase to the gain of 15 to 20 per cent. that our government experts consider both desirable and practicable. Perhaps the extra vegetables are growing in the Victory Gardens now.

Citrus Fruits and Tomatoes are grouped together on the ground of high vitamin C content which keeps well, and the fact that they are largely eaten raw and thus escape cooking loss. In justice to the vegetables of the cabbage family and to turnip greens, it should perhaps be said that several of these are so rich in vitamin C as to have a high content of this vitamin even after cooking. But, of course, they are less completely interchangeable with citrus fruits and tomatoes in meal planning. Oranges and grapefruit are richer in vitamin C than are tomatoes, while tomatoes have much higher vitamin A value than citrus fruits. Taken together this group just meets its quota, as illustrated in Table 1, on vitamin A value and thiamine, and falls below in calories, protein, calcium, iron, and riboflavin; but at only 6 per cent. of the cost of the entire dietary the citrus fruits and tomatoes furnish 35 per cent. of the vitamin C. Present levels of production and consumption of citrus fruits are considerably above those of a few years ago and no great further increase is expected under war conditions. Victory gardens may well continue the increase in the production and consumption of tomatoes.

Other vegetables and fruits may increase through greater attention to farm and family gardens even during the war; while as soon as transportation and supplies of materials for processing become normal the growing of vegetables and fruit for both fresh market and processing will probably increase rapidly to 20 per cent. or more above present levels. The land required for such an increase is practically negligible compared with the acreages of cotton and of the major food and feed crops.

* To be strictly logical the group should include yellow fruits and exclude yellow turnips; and sweetpotatoes belong here as well as in the group above.

Milk and its Products Other than Butter.—As all adequate studies of the subject have shown, and our agricultural economists have repeatedly emphasized in their discussions, milk is by far the most efficient livestock source of protein, so that the best use of the feed-grain supply is to direct more of it into the production of whole-milk products. Also, in addition to this outstanding efficiency of the milk cow among livestock transformers of feed protein into human food, she is also incomparably superior to other classes of livestock in the efficiency with which she concentrates from her feed and transmits to the human family both calcium and riboflavin—the two nutrients that in American dietaries most frequently fall below the Recommended Allowances of the National Research Council.

While handicapped by lack of exciting flavor or traditional association with feasting, the place of milk in the food supply grows steadily with scientific knowledge. Consumer demand increases through well-founded conviction of the importance of liberal use of milk as food throughout life. In the United States the resulting consumption of milk has grown with the increase and diffusion of the newer knowledge of nutrition; and production was about 119 billion pounds in 1942, but fell slightly in 1943. For 1944, the practicable goal under the existing economic conditions and shortage of feed grains in dairy regions was thought to be 122 billion, though productive capacity was estimated at 128 billion, with the possibility of increasing this rapidly to at least 143 billion pounds a year if, as a leading dairy expert said, we "really went out after it"; *i.e.*, if the conditions retarding the desirable increase of production were systematically attacked.

Undoubtedly the most serious of these retarding conditions has been the drawing away of too large a proportion of the feed-grain supply from milk production into pork production. Efforts are now being made to correct this, chiefly by such readjustments of price relationships as will make feed grain more available to dairy farmers.

Through attention both to breeding and to feeding, the production of milk per cow was increased, gradually but steadily, from 1934 to 1941, then it stood still in 1942, and fell somewhat in 1943, doubtless because in these two years the high price of pork was drawing grain away from milk production to pork production. Experts of the U. S. Department of Agriculture agree that by far the most important problem is adequate provision for getting existing feed grains into dairymen's hands.

As Auchter and Hambidge (3) have emphasized, it is becoming possible—by better breeding, care, feeding, and disease control—to have much more milk per cow than American agriculture has made hitherto. The 26 million dairy cows in the United States of last year produced an average of about 4,700 pounds of milk per annum. At the same time, the 800,000 cows represented by our dairy herd improve-

ment associations averaged 8,000 pounds of milk; the experimental herd of the U. S. Department of Agriculture at Beltsville, Maryland, averaged about 16,000 pounds; and the world's champion milk cow has produced 41,943 pounds of milk in one year. Production per cow is being improved by the use of superior sires and artificial insemination, as well as by better care and feeding.

It is of interest that the connotation of the term whole milk has changed with the growth of our understanding and appreciation of the mineral elements, especially the calcium, and the water-soluble vitamin, riboflavin. Both these are often the critical factors in American dietaries or food supplies; and of both, milk is the outstanding food source. Hence, whereas milk was formerly called whole for possessing its fat, it is now called whole for having its non-fat solids.

The percentage of the non-fat milk solids production which has been brought into human consumption has increased from 49 in 1933 to about 60 in 1943. The further increase of this fraction will depend largely on increasing total milk production so as to support milk-drying plants in more communities. Best use of our food and feed resources here calls *both* for more milk production *and* for its more effective direction into human consumption—its better utilization by man.

Nutritionists and agricultural economists now combine in urging promptness of action to increase milk production as being of the utmost importance. Table 1 shows high contributions of milk to the dietary as compared with its cost.

Eggs are rich in protein, iron, and vitamins. Next to whole-milk products, they are the most efficient source of animal protein.

Large quantities of dried eggs have lately been shipped overseas because they occupy little space in proportion to their food value.

Both poultry and egg production have risen very rapidly in the early war years and are now (1944) pressing heavily upon the available supplies of grain and other concentrated feeds.

According to customary computations the laying hen, while not as efficient as the milk cow, is more efficient than the meat animal in the transforming of feed nutrients into food for man. But, as Maynard has especially pointed out, the hen competes more directly with man in what she eats than do some of the meat animals (8).

Meat, including Fish and Poultry.—The most recently available official estimates at time of writing show per capita consumption of all meats, including fish and poultry, as follows: In the United States, 169 pounds a year; in Canada, 160 pounds; in Great Britain, 125 pounds. All these are very liberal amounts compared with the meat supplies of most other countries.

In this country beef cattle numbers have moved in cycles of about 8 years of increase followed by a shorter term of decrease. When the war began our beef cattle industry was on the upsweep of a marked

cycle and even the increased demand for beef did not bring enough to market to check the rise in numbers. In 1943 the ranges were overstocked and beef cattle numbers were exceeding the resources for feeding them. Late in 1943 marketing began to increase, and this has continued gradually, but in mid-1944 there was still risk of disaster from too many beef animals and too little to feed them. In order to avoid an acute shortage of feed about 5 million more cattle should be slaughtered in each of the years 1944 and 1945 than were slaughtered in 1943. Thus the needed retrenchment will mean actually more beef than before for perhaps 2 years or so; then, if well managed, there should be stabilization at a level approximating the middle instead of the present top of the "cattle numbers cycle." Also the beef cattle should come to the butcher without such heavy grain feeding as has often been practiced in the past.

Large savings in feed could be achieved by a combination of (*a*) confining grain feeding to that needed for the attainment of "good" slaughter grade, and (*b*) feeding less than a full feed of concentrates during all or part of the finishing period and increasing the quantity of roughage fed. After a steer has a moderate finish, additional gains consist chiefly of fat—fat in excess of what is required to make the beef attractive. Even the fattening steer works for us more efficiently if he is not too fat. In part, this is explained by the fact that large and relatively thin cattle can utilize to good effect large amounts of pasturage and other bulky fibrous feeds in the eating of which they do not compete for anything that people eat. Thus, up to the point at which they begin to compete for grain and fine-quality tame grass, the beef animals are an asset; but beyond that point they rapidly become an increasing liability from the standpoint of our efficient use of our food-production resources. It might help in eliminating the practice of wastefully heavy grain feeding of steers if we could agree to drop the market designations which profess to stand for any quality higher than Good. Scientifically and patriotically, Choice and Prime are really not so good. They represent the product of a too artificial process.

Numbers of cattle on Western ranges are now (1944) near the maximum attainable after long seasons of exceptionally favorable grazing conditions. Weather less good for grass may, unless the numbers of beef cattle are soon reduced, result in such malnutrition of cattle on ranges as seriously to impair their value to both producer and consumer. Fewer will be better, and the prompt marketing of those in excess of best numbers for the ranges and feed-lots will bring the extra beef to consumers at a time of high appreciation and purchasing power.

In the Corn Belt, our agricultural economists recommend, the numbers of beef cattle may best be adjusted largely to the available roughage feeds, corn stalks, silage, and oat straw, supplemented by

those fractions of the local grain crops that do not reach such maturity as to be suitable for storage, together with such minimum quantities of feed grains as are required for an efficient ration.

For best economy of resources, beef animals should spend as much as possible of their lives on the ranges where they do not compete with other livestock for crops grown on cultivated land. The condition of the forage on the open ranges thus becomes an important factor in our food economics.

About 8 years ago it was estimated that range lands were producing only half as much as they might. Since that time the productivity of the ranges has been improved, but doubtless is capable of much further improvement, with corresponding increased production of beef, lamb, and mutton with little if any feeding of grain or other cultivated crops. It is thought that a more scientific management of the cattle that range at will over neglected areas would considerably increase the supply of forage-fed beef, or would permit a grain-fed finish to be attained with expenditure of much less grain. Range grazing seasons can be set with reference to seasonal abundance of the native forage, then systematically supplemented by improved pasture and supplemental feeding; and beef outputs thus materially increased with minimal expenditure of grain.

The Southern States, on the other hand, by increasing their peanut crop, as recommended, will probably augment their supply of peanut hay sufficiently to enable them to handle successfully their recently increased numbers of beef cattle. It may surprise readers of western literature to know that the South, even without counting Texas and Oklahoma, is estimated to have as many as 17 per cent. of the cattle now (1944) in the United States. Southern regions hitherto afflicted with inadequate food supplies and largely-idle land can thus serve themselves and the nation well by growing larger crops of peanuts and feeding the peanut hay to moderately increased numbers of local cattle.

In all dairy regions the culled milk-cows and the majority of calves, whether as veal or young beef, constitute a steady contribution to the meat supply.

How to have a national production of the maximum supply, either of beef or of total meat, that will not draw too heavily upon the resources needed for optimal supplies of other foods—with good balance between classes of livestock and between animals and feedingstuffs for them, with orderly movement of meat animals to market at the ages and weights representing best use of resources, is a complex problem in management which is now being constantly and comprehensively studied in Washington by what is doubtless a larger, better informed, and more open-minded body of experts than has ever before been continuously assembled for such a purpose.

Pork can undoubtedly be obtained with best economy and efficiency from the animals and the feed now in sight in the United States by marketing hogs from now on at least 20 pounds below the average market weight of 1943. Certainly at least this much reduction of market weight should be planned for. Perhaps a still lower average market weight would be better in that it would set free more grain for use in milk production. The *less* heavily grain-fed the hog, the higher the percentage of protein he brings the consumer; and this protein is better utilized than is extra fat in retail pork. The extra fat for commercial use as such is better produced through soybeans and peanuts than through hogs.

All who have adequately considered the subject are now agreed upon the policy of adjusting the nation's meat animal population to a sounder relationship with feeding resources, so that we may carry more adequate grain reserves at all times.

The Secretary of Agriculture wrote officially in his annual report that in 1943 "farmers saved 74,050,000 spring pigs, or 22 per cent. above the number saved in 1942 and 37 per cent. above the number in any previous year. They appear to be saving about 21 per cent. more fall pigs than in 1942. In fact, the 88 million hogs that will be on farms January 1 will overtax the feed supply." Accordingly, production goals and support prices for 1944 were so set as to induce less feeding of grain to swine. This will result in a lessened production of lard; and only later, a somewhat lessened amount of pork will come to market. The first effect of a policy of earlier marketing of hogs was, of course, to increase the total number marketed in the winter and spring of 1943–44, so that in May 1944 the nation's facilities for storage of pork were overtaxed and its rationing was suspended in order to move it more rapidly to consumers.

In the permanent view, we should regard the extraordinary increase of annual pig crop in the United States from about 80 million in 1940 to 85 million in 1941, 105 million in 1942, and 122 million in 1943, as an effective but temporary emergency measure. There was an abnormally active market for meat, and swine were also the quickest means of meeting the war emergency shortage of fat. Now that the needed fat is being largely obtained from oil-seeds, and grain is so urgently needed for milk production, we must readjust a swine population that had jumped about 50 per cent. while the human population and its foster-mothers, the milk cows, were increasing about 5 per cent.

We have now made sufficient progress with our fat-supply problem so that we can and should take the next logical step and free our food-production resources from the burden of supporting so many swine.

If we were fully satisfied with the nutritional status of all our people we could maintain our other food supplies as they are by a very

small percentage decrease of our pork production; while if the adjustment is made a trifle more wholeheartedly, it will react to bring to our lower-income people a larger measure of the benefits of the newer knowledge of nutrition.

The Supply of Fats.—Production of vegetable fat in the United States was about one-half greater in 1942–43 than the average for 1937–41.

Soybeans and peanuts are doubtless the two largest potential domestic sources of direct-crop fat for our national food supply. And they do not directly compete for the same land to any material extent, peanuts thriving best in more southern regions and on more sandy soils than are best for soybeans.

The growing of cotton can make a good use of resources up to the acreage whose fiber crop will be in good demand, but to expand cotton beyond the demand for its fiber is not as efficient a way to get extra protein and fat as to raise these in the form of soybeans and peanuts, especially when these latter crops are expanded only in the areas well suited to them.

In the expectation that its cotton acreage will change relatively little, we may expect the South to increase markedly its production of sweetpotatoes, peanuts, and crops for feeding to local livestock, including poultry.

Flax already occupies almost as much of our land as is well suited to the growing of flaxseed, so we do not look to this as an importantly expandable source of fat and of feeding-stuff in the United States.

Our present discussion relates chiefly to soybeans and peanuts.

The expressing of the fatty oil from any of the oil-seeds leaves a press-cake which is rich in proteins of high nutritive value either for human nutrition or for the feeding of farm animals.

Hence, increased production of any of the oil-seed crops helps the food situation both as to fat and as to protein supply.

The yield of soybeans in bushels per acre is usually around half that of corn. Man-labor requirements are somewhat less for the soybeans. The mature soybeans are much richer in oil than corn, the beans yielding about 15 per cent. of their weight of oil. The market for the oil is thus a very large factor in determining the farmer's preference between the two crops. Thus, corn and soybeans compete rather directly in production. More beans, more oil and more total protein; more corn, a chance of more money through pork.

As a matter of fact the first years of the Second World War brought forth from American agriculture an increased production of corn, and through corn, of pork and lard; but swine and poultry were allowed to multiply so greatly that by mid-1943 they had devoured dangerously near all of our grain reserves so that it became important to shift somewhat from lard to seed-oils and so increase the acreage of soybeans

relatively more than of corn, though the total corn acreage was still at least 6 or 7 times that of soybeans.

Sugar may, perhaps, produce more food calories per acre than other crops; but it needs to be remembered that, nutritionally, the calories constitute the sole contribution that sugar makes to the food supply. So sugar should not exceed the amount that can and will be balanced by all the specific nutrients in the full quantities that we need for our best health and efficiency. These "balancing" nutrients to be had from other foods include eight specific amino acids from the proteins of our food, and each of the nutritionally essential fatty acids, vitamins (or provitamins), and mineral elements.

Or, we may put the thought in a slightly different way. The present American per capita consumption of sugar probably exceeds that of a century ago by an amount that furnishes at least a tenth of our total food calories. If we had not thus increased our sugar consumption we would presumably now be getting about one-tenth more of each essential nutrient than we are. Are our margins of insurance with respect to every nutrient so liberal that a tenth more or less is negligible? The rationing which has reduced our consumption of sugar is not reported to have caused nutritionists any anxiety on this account.

In final summary, present production goals aim to reduce, somewhat, the amount of grain which is being turned into fat meat, in order that more grain may be turned into bread or breakfast cereal and into milk.

Grain turned into milk yields a much greater nutritional return than that turned into meat, and the milk is much more efficient than the meat in supplementing and balancing the direct food crops.

Even if our direct domestic use of grains and potatoes as human food remained unchanged, a higher relative proportion of milk is nutritionally important, because it is the outstanding source of calcium and riboflavin, the two nutrients in which our dietaries most often fall below the Recommended Allowances.

How far can we see the probabilities in other countries?

So many people will have a part in deciding what foods to send, in what quantities, to what countries, that it is impossible to foretell these decisions. We can, however, prepare for the emergency by adjusting our production and consumption in the directions indicated by the Department of Agriculture goals and explained in the foregoing discussion.

We can raise more of the direct food crops with emphasis on wheat, beans, peas, oil-seeds, green and yellow vegetables, and tomatoes.

We can conserve grain by marketing meat animals at somewhat earlier ages and with much less of heavy grain feeding; and also by making good use of pastures and forage of all kinds.

We can each, through consumer demand, take daily action to make good the principle that our livestock population be adjusted to a fuller

investment of feedingstuff resources in the animals that give best returns from the standpoint of the human food supply.

This would mean a continuingly increased prominence of dairy products other than butter and of the more perishable kinds of fruits and vegetables in our dietaries, with resulting enhancement of the nutritional wellbeing and working efficiency of our people.

It would mean, too, that we should at all times be ready to send overseas large amounts of grain or flour and dry beans or peas, and moderate amounts of meats and fats. These foods are all practicable to transport and distribute. Together they meet the most prominent needs and desires of the peoples of Europe. And they are foods of which we can ship important quantities while at the same time continuing to improve the nutrition of our people.

Adjustment of our meat-animal population so that we may carry safer reserves of grain, and so that our feedingstuff resources shall be invested more largely in the classes of livestock that return the best values for human nutrition, is clearly, and rather urgently, desirable whether we contemplate considerable shipments of meat overseas, or not.

If we contemplate considerable shipments of meat, we should use our meat packing and storage facilities at full capacity so as to be able to ship enough in time whenever and wherever it will do the most good.

If we do not contemplate such shipments we may continue our present (June 1944) high level of meat consumption for some time, to be followed by adjustment to a more stable equilibrium at an only slightly lower level later.

In any case we should be willing to have another period of somewhat closer rationing of meats, fats, or both, as may be involved in reaching a more stable world food situation.

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Scientists Find "Insect Trapper" Fungus.—The discovery of the first fungus known to trap and destroy insects is announced by the Agricultural Research Administration, U. S. Department of Agriculture. It is a microscopic soil-inhabiting fungus, said to be "unmistakably adapted to prey primarily on insects" and "under natural conditions presumably is given wholly to a predaceous mode of life."

According to the scientist who made the discovery, Dr. Charles Drechsler, pathologist of the Plant Industry Station, Beltsville, Md., there are more than 60 fungi known to capture animal organisms, such as amoebas, nématodes, rotifers, and other small forms, but the new one, called by the discoverer *Arthrobotrys entomopaga*, (the second part meaning "insect trapper,") catches springtails, insects that jump by means of springy near-end spines. They are found frequently on decaying vegetable matter and are known sometimes to attack economic plants in the growing stage as, for example, alfalfa in Australia.

The fungus is of the type that sends out horizontal threads or hyphae that crisscross and actually join at the intersections to make a network that may be likened to a piece of electrically welded wire fencing in miniature. This network sends up columns that exude at the tops droplets of a clear adhesive liquid, the death trap for the springtails. Once caught, the insects are run through with a cobweb of filaments that transfer the food elements from their bodies to the fungus.

Although Dr. Drechsler has given this new species of *Arthrobotrys* the name of insect trapper, he does not think it likely that this is the only fungus that lives in this way.

He suspects there are others that likewise prey on minute insects inhabiting porous decaying materials. He believes that some of these fungi, as for example those attacking nematodes, may be a factor in protecting plants against parasites.

R. H. O.

THE INFLUENCE OF PHOTOGRAPHIC DEVELOPERS CONTAINING HYDRAZINE UPON THE CHARACTERISTIC CURVES OF PHOTOGRAPHIC MATERIALS. I.

BY

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Communication No. 974 from the Kodak Research Laboratories.

INTRODUCTION.

Hydrazine and many of its derivatives have been known as developers of the photographic latent image since the publications of Andresen.[1,2] Furthermore, the use of hydrazines as halogen acceptors to decrease solarization in photographic emulsions has been described in the literature.[3] This paper presents the evidence of a new phenomenon occurring under certain conditions during the development of silver halide emulsions in the presence of various hydrazine derivatives. The phenomenon is characterized by increases in the inertia speed, $1/i$, and contrast (gamma) of the photographic materials studied as compared with those of normal control development.

DESCRIPTION OF THE PHENOMENON.

It was observed by two of us (A. P. H. Trivelli and W. F. Smith) in 1934 that when certain hydrazine derivatives, e.g., semicarbazide hydrochloride and hydrazine dihydrochloride, were added to some gelatino-silver halide emulsions before coating on a suitable support, they produced significant increases in speed and contrast, as shown by test plates exposed sensitometrically and developed in a process-type developer, such as Kodak D-8, and compared with coatings of the emulsion without addenda given similar processing treatment. Subsequently it was established that similar or identical effects could be obtained by introducing the hydrazines into the developer solution in appropriate but small quantities. More recent studies of this phenomenon have elucidated many interesting points concerning the mechanism of action of the hydrazines in photographic developers, although the data at present are insufficient for a complete explanation of the observed facts. Table I demonstrates the magnitude and character of the effect for an arbitrary experimental arrangement.

A number of facts have been accumulated concerning the conditions

[1] Andresen, M., Phot. Mitt., **28**, 286; 296 (1892).

[2] Kirby, J. E., U. S. Pat. 2,220,929.

[3] Mees, C. E. K., "The Theory of the Photographic Process," pp. 273–274, The Macmillan Company, New York, 1942.

TABLE 1.

Recordak Film.

Exposure: 1/25 sec. Eastman Ib sensitometer, tungsten 3,000° K.

Experiment	Developer Composition	Speed (1/i)	Gamma	Fog	Toe [4]
C89-20b	Kodak D-8 (control)	2.34	2.4	.04	.61
C89-20c	D-8 + 0.03 g./1,000 cc. $N_2H_4.2HCl$	6.76	8.0	.05	.16

necessary to produce this action with hydrazines. Foremost appears to be the observation that, in order to obtain the effect, the hydrazine must be present simultaneously with a normal photographic developing agent. Thus, bathing of photographic materials with hydrazine solutions, either before or after exposure, followed by thorough washing and subsequent development results in no such effect. On the other hand, bathing of a sensitized material with a hydrazine and development without washing will yield a strong effect. Between bathing and development, the film or plate may be dried in air.

A further requisite for obtaining the phenomenon concerns the kind of photographic developing agent used, which, according to all the data obtained thus far, must belong to the class of organic developers containing one or more phenolic groups, e.g., *p*-hydroquinone, *p*-aminophenol, *o*-hydroquinone (catechol), or methyl-*p*-aminophenol.

TABLE 2.

Semicarbazone	Per Cent. Contrast Gain over Control	Rate Constant of Dissociation ($K_1 \times 10^5$)	Equilibr. Const. of Dissoc.
Benzaldehyde	−11	.62	.32
Pinacolone	+ 4	86.00	1260.00
Acetaldehyde	+18	1040.00	2.90
Cyclohexanone	+25	7600.00	214.00

Extensive experiments have demonstrated that various hydrazines produce the phenomenon to a varying extent. These include alkyl and aryl hydrazines substituted on one nitrogen atom only, as well as hydrazides of mono-, di-, and polybasic-aliphatic or aromatic acids, and hydrazides of carbonic acid (semicarbazides). In a few cases, the derivatives obtained by condensing hydrazines or semicarbazides with aldehydes (hydrazones, semicarbazones) have also produced the effect. For a series of semicarbazones, the order of increasing activity is benzaldehyde, pinacolone, acetaldehyde, cyclohexanone; this sequence corresponds to the order of rates of dissociation at 25° C. and pH 7.0 given by Conant.[5]

[4] "Toe" is arbitrarily defined as the log *E* range required to increase the density from 0.15 above fog to 1.5 above fog.

[5] Conant, J. B., "Equilibria and Rates of Some Organic Reactions," p. 24, Columbia University Press, New York, 1932.

The appearance of images that have been developed in the presence of hydrazines to increase contrast and speed may be characterized to a varying extent by a graininess pattern which in these Laboratories we have termed "wooliness." This graininess is visible in medium density areas with the aid of low-power magnification. The usual graininess comparisons made with controls at greater magnification do not show any increase in graininess. With some photographic materials, when an excessive effect is present, a granular type of fog appears, which consists of round dots of high density varying from a size just visible by low-power (10×) magnification to large dots plainly visible to the naked eye. This type of fog we have termed "pepper." Comparisons of individual, developed grains of a process-type emulsion with the grains of the same emulsion developed in a hydrazine-containing D-8

FIG. 1

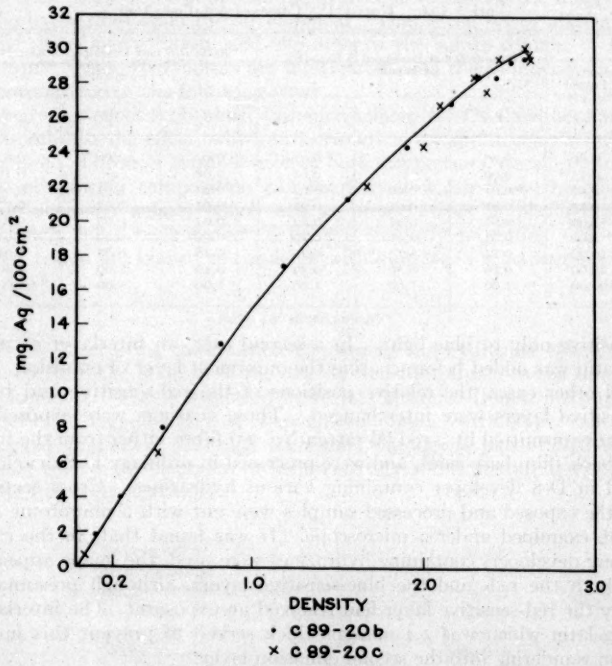


FIG. 1. Mg. silver/cm.² *vs.* photographic density.

developer showed no differences when examined with an electron microscope. Fig. 1 shows that the photometric equivalent is unaffected within the limit of experimental error by the presence of hydrazine in the developer. The data plotted in Fig. 1 are for the experimental strips reported sensitometrically in Table 1.

From these data it seems clear that the gain in density obtained for a given exposure must be due to a larger number of developed grains. This gain might be caused by the development of grains carrying a latent image which is too feeble to develop under ordinary processing conditions or might be due to fogging of unexposed grains. Our experiments show conclusively that the latter is the case.

In one experiment, a chlorobromide process-type emulsion sensitized only to blue and red light was coated on a film support; this layer of emulsion was then overcoated with another layer of the same emulsion

TABLE 3.

pH = 10.6. T = 65° F. Constant agitation—tank.

Percentage of Fogged Grains Added	Ratio *D*/*D* max Developed in Exp.-D-1 (in minutes)			Ratio *D*/*D* max Developed in Exp.-D-1 + $N_2H_4 \cdot 2HCl$ (0.50 g./1,000 cc.) (in minutes)		
	5′	10′	20′	5′	10′	20′
0.00	0.00	0.00	0.01	0.00	0.00	0.02
0.25	0.00	—	0.01	0.00	—	0.10
0.50	0.01	0.00	0.02	0.01	0.01	0.68
0.75	0.01	0.01	0.02	0.01	0.02	0.77
1.00	0.01	0.01	0.02	0.01	0.03	0.83
5.00	0.04	0.06	0.10	0.06	0.69	0.91
10.00	0.11	0.13	0.20	0.18	0.87	0.91
25.00	0.26	0.30	0.39	0.46	1.00	1.00
100.00	1.00	1.00	1.00	1.00	1.00	1.00

sensitive only to blue light. In a second case, an interlayer of plain gelatin was added before coating the outermost layer of emulsion. In still other cases, the relative positions of the red-sensitive and blue-sensitive layers were interchanged. These coatings were exposed to light transmitted by a red (Wratten No. 25) filter, either from the front or back (film base side), and were processed in ordinary D-8 developer and in D-8 developer containing various hydrazines. Cross sections of the exposed and processed samples were cut with a microtome and were examined under a microscope. It was found that, in the cases where developers containing hydrazines were used, the image appeared in both the red- and the blue-sensitive layers, although presumably only the red-sensitive layer had received an exposure. The interlayer of gelatin which was 2.4 microns thick served to prevent this image from wandering into the second emulsion layer.

Another experiment was made along the lines of an earlier experi-

ment described by Renwick.[6] In this case, a quantity of a slow chlorobromide emulsion was split into two portions. One portion was fogged by passing the melted emulsion over a sloping glass plate between exposing lamps at such a rate that the print-out effect was minimized and the developable density in D-8 developer was at a maximum. Varying percentages of the fogged portion were mixed with the unfogged emulsion and coated at a constant thickness on a film support. These coatings were processed in an experimental alkaline hydroquinone developer, Exp.-D-1, and in the same developer containing hydrazine. The data are given in Table 3 and show the extent of fogging of previously unexposed grains by development in the presence of hydrazine.

INFLUENCE OF DEVELOPER COMPOSITION.

Two factors besides the structure of the organic developing agent have been found to be of special importance in regulating the influence of the hydrazines upon the development. One is the pH or alkalinity of the developer solution, and the other is the sulfite content of the solution. These two factors are interrelated, and their influences may be summarized in the following way:

A greater effect is obtained with increasing pH. On the other hand, sulfite inhibits the effect, which is decreased as the sulfite concentration increases. Through manipulation of both components, developer solutions of varying composition can be derived which show the effect. Other chemical agents which act as preservatives in photographic developers have been found to behave similarly to sulfite. All the agents tested are known to condense with quinone in a manner similar to sulfite.

RATE OF DEVELOPMENT.

Some interesting information concerning the hydrazine effect was derived from the relation of the density and the gamma to the time of development. Fig. 2 is plotted from data obtained from a mixture of 10 per cent. fogged emulsion (slow chlorobromide) with 90 per cent. of unfogged emulsion. Curve B represents the log-log plot of D/D max against time of development for a film coating of this mixture developed in the following developer:

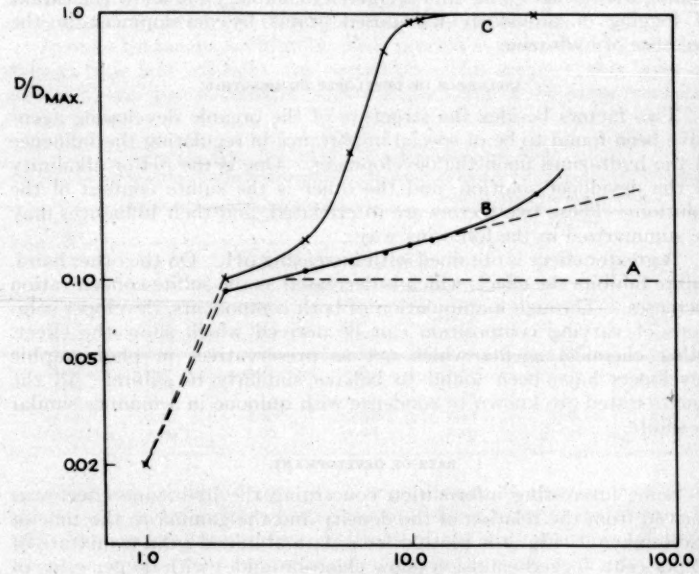
Exp.-D-1.

Sodium sulfite	20.0 g.
Hydroquinone	30.0 g.
Potassium bromide	5.0 g.
Kodalk	40.0 g.
Sodium hydroxide	4.0 g.
Water to	1,000.0 cc.

[6] Renwick, F. F., J. Soc. Chem. Ind., 39, 157T (1920).

Curve C, Fig. 2, represents similar data for an identical developer with 0.50 g. per 1,000 cc. of hydrazine hydrochloride added. Curve A, Fig. 2, represents the ratio of D/D max expected if D were a linear function of the percentage of fogged emulsion added to the mixture. Comparison of Curves B and C indicates that the initial kinetics of development in the absence and presence of hydrazine are very similar. As development continues, however, the hydrazine shows its effect in a

FIG. 2



DEVELOPER SAME AS IN FIGURE 1

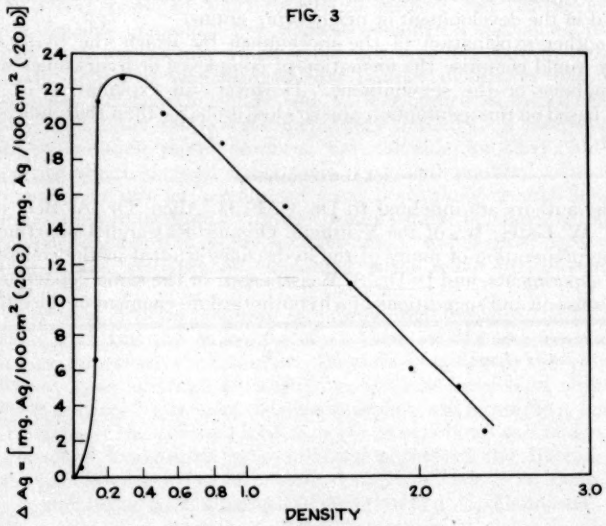
steeply ascending curve which diverges from that of the comparison development and in a relatively short time reaches D max.

Figure 3 is a plot of the increase in silver or density of a particular sensitometric exposure against the density of the same exposure on development in D-8 developer and in D-8 developer containing 0.03 g. per 1,000 cc. of hydrazine dihydrochloride. These data for a slow chlorobromide emulsion show the rapid increase in the effect with increasing density. This gain eventually passes through a maximum

and diminishes to zero at high exposures or densities of the control, obviously because in this region the silver halide becomes exhausted.

All these results indicate that the influence of the hydrazines upon the characteristic curves of photographic materials depends upon their ability to induce development of unexposed or weakly exposed grains in the vicinity of exposed and normally developing grains. Since this acceleration in development or local fogging action is not a normal property of either the hydrazines or polyphenolic developers used alone, it seems probable that the fogging agent is produced during the course of the regular developing reactions. This agent is possibly the product

FIG. 3



of a reaction of the hydrazine with the phenolic developing agent or with an oxidation product of the latter. The effect seems to be dependent upon the kinetics of various reactions of the system, since we have not succeeded in obtaining similar effects by the use of oxidized developers (quinones) and hydrazines. There is some reason to believe that even in the absence of hydrazines similar mechanisms of development enter into developments carried out with alkaline hydroquinone (process-type) developers, although to a considerably less extent.

As a hypothesis to account for this phenomenon, we are tentatively suggesting the formation of a free radical semiquinone which is considered to be the fogging agent. Such semiquinones are well known in

the case of hydroquinone-quinone oxidation reductions and are favored by high alkalinity.

The effect of sulfite suggests the explanation that it removes the semiquinone. This may take place either directly or by a reaction with the quinone with which the semiquinone is in equilibrium. The formation of hydroquinone sulfonate from sulfite and quinone is a well-known reaction.

The active agent, i.e., the semiquinone, is probably formed as an intermediate in the reduction of the respective quinones to the hydroquinone. This reduction may occur at the surface of unexposed grains where hydrazine is adsorbed and where quinone diffuses which has been formed in the development of neighboring grains.

Another explanation of the mechanism by which the hydrazine works would comprise the formation of complexes or hydrazones with the quinone or the semiquinone. However, an explanation of the effect based on this assumption appears less probable than that indicated above.

ACKNOWLEDGMENT.

The authors are indebted to Dr. C. F. H. Allen, Dr. A. Bell, and Dr. J. W. Gates, Jr., of the Synthetic Organic Research Department, for the preparation of many of the hydrazines studied in the course of these experiments, and to Dr. A. Weissberger, of the same department, for discussion and suggestions of a hypothetical mechanism of the effect.

ROCHESTER 4, N. Y.,
February 24, 1944.

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NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

POWDER METALLURGY.

Powder metallurgy is the art of forming metal objects by compression and sintering of metal powders instead of by conventional practices of melting, casting, hot and cold working, and machining to shape. Its first practical application was in the fabrication of tungsten, the high melting point of which precludes melting and casting in any known crucible or mold. For years it remained a specialized process limited to a few metals which could not be fabricated by ordinary methods. Later it was realized that powder metallurgy has useful applications for common metals, and the development of special bearings greatly stimulated the powder metallurgy industry. These bearings, made of common metal powders, are valuable for their "oil-less" properties which depend upon their controlled porosity. Other developments in powder metallurgy have been along several lines, for example, the production of "alloys" which cannot be made by foundry methods (tungsten-tin, copper-graphite, etc.), and of small parts which require undue amounts of machining when made by conventional methods.

In general, articles produced by powder metallurgy are individually small in size, but the aggregate of all these small parts, particularly under the impetus of the war effort, amounts to an appreciable industry. However, most of these successful applications have been developed by "cut and try" methods; fundamental data are incomplete, and the interrelation of the different factors in the process is not well understood.

Personnel limitations have hitherto prevented the Bureau from conducting investigations in powder metallurgy, but work along these lines is now being started under the direction of J. G. Thompson. The initial work will consist in a study of commercial metal powders to determine for each powder its characteristic average size and shape, the particle size characteristics as determined by various means, and the density and other pertinent properties. Information on the effect of temperature and pressure in the compacting operation is needed, and likewise the effects of time, temperature, and atmosphere of the sintering operation. Individual metal powders will be studied at first; mixtures of 2 or more metals and mixtures of metals and nonmetallic powders will follow.

In preparation for the experimental work, a supply of metal powders and some necessary equipment have been purchased, other equipment

* Communicated by the Director.

has been and is being designed for shop construction, and the literature is being reviewed for available information on powder metallurgy.

SOIL CORROSION OF PIPE LINES.

In 1922, the Bureau started a study of the corrosion of pipe lines in typical soils throughout the United States. The results obtained early in this investigation showed that in many soils the damage by corrosion was much greater than had been anticipated, and that no ordinary pipe material could be expected to give satisfactory service under these conditions. In order to improve this situation, manufacturers of pipe materials commenced the development of products suitable for use in corrosive soils, and about 1932 the Bureau began a supplementary investigation intended primarily to assist manufacturers in determining how well the problem had been solved.

Instead of attempting to secure all the specimens that would be needed if systematic data on the effect of individual alloying elements or their amounts were to be obtained, manufacturers were invited to submit materials on which they desired information. Furthermore, the specimens were buried under simulated practical conditions so that accurate technical control was not feasible. Also, since very few specimens of each type were included, dispersion of the data resulting from the general conditions of the tests makes it impossible to draw very definite conclusions with regard to the comparative merits of the several materials investigated. Nevertheless, the data do give a general indication of what may be expected of a considerable variety of alloys when exposed to severe soil conditions. The addition of very small percentages of alloying elements does not have a marked effect on the rate of corrosion, but rather large percentages are apt to bring about a considerable improvement in corrosion resistance.

Steels high in nickel and chromium, and copper alloys high in copper, are very resistant to nearly all soil conditions. Lead corrodes only slightly after it acquires a coating of an insoluble lead salt.

Bursting and crushing tests indicate that asbestos-cement pipe gained somewhat in strength during exposure to the soil for 4 years.

A 3-ounce coating of zinc added about 3 years to the life of steel exposed to some of the most corrosive soils in the test sites. Lead coatings appear to be inadequate for severe soil conditions.

Several thick experimental coatings prevented practically all corrosion at most of the test sites for 9 years. Air-dried bakelite coatings blistered within 4 years, and a baked-on bakelite coating blistered in a few places; otherwise it showed no change in appearance after this length of service. Pitting occurred under some of these blisters. Most thin coatings blistered, became brittle, and permitted rusting and pitting of the metal to which they were applied within 4 years.

The latest progress report on the soil corrosion work will appear as RP1602 by Kirk H. Logan and Melvin Romanoff in the September number of the Journal of Research.

ANCHORAGES FOR REINFORCEMENT IN CONCRETE.

In designing reinforced concrete structures, it is ordinarily assumed that the tensile and compressive deformations in the steel reinforcement are the same as in the adjacent concrete. To assure an approximation of this common action of the two materials, the design stresses in the reinforcement and the bond stresses between the concrete and the reinforcement are limited to values indicated to be safe by the results of laboratory tests and experience. Often, the necessary anchorage for the reinforcement is obtained by bending the ends of the bars to form semi-circular hooks. Current specifications limit the radii of such hooks and their lengths beyond the point of curvature but they do not include effective requirements governing the surface texture and the nature of the deformations on deformed bars. In an investigation of the factors that influence the effectiveness of hooks as anchorages for reinforcement, it has been found that the deformations on the bars have a greater effect upon the performance of hook anchorages than any of the factors now limited in specifications. The results of the investigation provide information on which to base specifications for anchorages that require less steel and are more likely to function as intended than those now in common use.

INSTRUMENT FOR DETERMINING AIRWORTHINESS OF FABRICS.

A test and calibration program has been conducted by the Bureau on a new mechanical fabric tester developed by the Civil Aeronautics Administration. This work has resulted in the refinement of the instrument which determines the airworthiness of doped fabric aircraft covering. It is expected that this mechanical method will supplant the method in which the inspector judges airworthiness by his ability to push his thumb through the fabric.

The instrument operates on the same principle as the automatic center punch. The foot of the instrument is held against the surface to be tested and a steadily increasing pressure is applied, thus compressing a "firing" spring. When the spring has been compressed a predetermined amount, it is released by an automatic trigger and drives a hammer against a plunger which impinges upon the surface being tested. The energy of impact was obtained by measuring the indentations produced in a block of lead.

In studying the instrument, test panels were prepared from different weights of cloth, both new and chemically deteriorated to known

extents, and coated with good, semi-brittle, and very brittle dope in different amounts. Impact tests were made on all panels while in equilibrium with air having a temperature of 73° F. and a relative humidity of 50 per cent., and on some panels while at temperatures of − 40° and 160° F.

The impact energy required to drive the plunger through cloth coated with good or semi-brittle dope at 73° F. correlated well with the breaking strength of the doped cloth. Doped cloths became stronger and more resistant to penetration by impact as the weight of the dope increased. Very brittle dope was readily detected by ringworming caused at low impact energy. Tests of fabrics at − 40° and 160° F. showed the value of the instrument for studying the changes in doped cloths with temperature. Cloths coated with both good and semi-brittle dope became very brittle at − 40° F. and were penetrated at low impact energy. The cloths coated with semi-brittle dope became pliable at 160° F., and more impact energy was needed for penetration than at 73° F.

Twelve of the instruments have been designed and built by the technical development division of the Civil Aeronautics Administration and calibrated in the textile section of the Bureau. These have been distributed to interested organizations in order to obtain comments from field inspectors with regard to the instrument and its use in detecting unairworthy fabric coverings on airplanes.

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FLOWMETER FOR OXYGEN REGULATOR TEST SETS.

The safety of aviators flying above 20,000 feet depends upon proper functioning of aircraft oxygen regulators. This can only be ensured by periodic checking with special testing equipment. At the request of the Bureau of Aeronautics of the Navy Department, the Bureau has developed a readily portable field test set so that this necessary checking can be carried out at any landing field. With the great increase in the number of planes in service there has been an urgent demand from both the Army and the Navy for these sets.

Accurate flowmeters to measure the rate of flow of oxygen, or oxygen and air mixtures, are essential parts of each unit. Originally a commercial flowmeter known as a "rotameter" was used, but when bids were asked for quantity production of the test sets it was found that the demand for the rotameters for other essential uses was such that deliveries of the sets might be delayed for months. Development work was, therefore, rushed on a flow meter of a new type which could be manufactured easily and quickly from non-critical materials, and which has proved to be even better for the purpose than the rotameters previously used. This "National Bureau of Standards Flowmeter" is simply a can of tightly packed glass wool through which the oxygen or

air flows. The pressure difference between the ends of the can, measured by a differential gage, is directly proportional to the flow. The final designs of this flowmeter were furnished to the manufacturer, and the field test sets in which it is included are being calibrated at the Bureau as rapidly as they are produced.

DRY CELLS AND THEIR RAW MATERIALS.

Because of the War, batteries have been put to new uses under extreme conditions. This has encouraged research on dry cells and their raw materials. Howard F. McMurdie of the Bureau has prepared a report for the Transactions of the American Electrochemical Society, that is concerned with the crystalline products formed during discharge of the cells and identification of the various MnO_2 minerals in the ores used as raw materials. X-ray and electron diffraction studies indicated that 5 varieties of MnO_2 or closely related compounds occur in battery oxides. These are pyrolusite, ramsdellite, cryptomelane, γMnO_2 and δMnO_2. The last two are poorly crystallized. Electron micrographs showed that the particle size and shape vary greatly between different oxides.

Prismatic crystals of $Zn(NH_3)_2Cl_2$ are formed in the paste of undischarged cells after some aging, while small hexagonal plates of $ZnCl_2.4Zn(OH)_2$ are found in the pastes of discharged cells.

After discharge, the manganese in a cell was found to be largely in the form of $ZnO.Mn_2O_3$ (heterolite). This indicates that of the oxygen in MnO_2 only one-fourth is available for depolarization in dry cells.

COMMERCIAL STANDARD FOR MARKING SILVER JEWELRY.

The National Stamping Act of 1906 sets forth tolerances for items marked "Sterling," "Sterling Silver," "Coin," "Coin Silver," or with the actual fineness; but it fails to set forth exemptions not to be included in any assay for quality, such as screws, rivets, springs, and other elements of construction that are generally considered by the trade as necessarily being of other materials in order to give adequate service in the hands of the user.

The words "silver" and "solid silver" have, as a matter of trade usage, come to mean sterling silver, unless otherwise indicated. In order that these practices and usages might become a matter of public record and acceptance on a nation-wide basis, the New England Manufacturing Jewelers' and Silversmiths' Association requested the Bureau's cooperation in establishing a commercial standard for marking jewelry and novelties of silver, exclusive of flatware, hollowware, and toilet ware.

As a result of these efforts, such a commercial standard has just

been released. It was developed, adjusted and accepted by all those directly concerned, including producers, distributors, users, and testing laboratories. It bears the title, Marking of Jewelry and Novelties of Silver, Commercial Standard CS118–44, and is available from the Superintendent of Documents, Government Printing Office, Washington 25, D. C., at 5 cents per copy.

The standard sets forth the accepted definitions, requirements for quality marks, recognized exemptions, and requires that, in order to fix responsibility, quality marks shall be accompanied by the name or trade mark of the manufacturer or seller, duly applied for or registered under the laws of the United States. Tolerances under the standard are the same as those provided by the National Stamping Act, and an excerpt of these tolerances is included as a matter of information.

THE FRANKLIN INSTITUTE.

(Proceedings of a special meeting held Wednesday, September 13, 1944.)

PRESENTATION OF ORDNANCE DISTINGUISHED SERVICE AWARD.

A special meeting of The Franklin Institute was held at 4 o'clock in Franklin Hall, The Franklin Institute, Mr. Charles S. Redding, President, presiding.

After the playing of the National Anthem by Lieutenant Guy Marriner, Mr. Redding extended greetings to members, employees and friends of The Franklin Institute as follows:

"This is a special and very significant meeting of The Franklin Institute. I am very happy to welcome our distinguished guests, our members and friends. This afternoon The Franklin Institute is to be honored for the outstanding work done within its walls in the field of Ordnance Research and Engineering. We are taking part in an historic occasion in the annals of the Institute—an occasion signifying the continuing and outstanding value of The Franklin Institute to our country and its scientific progress.

"The man who has led the Institute and its staff in the prosecution of the job of finding solutions to the problems which it has been called upon to solve is our Secretary and Director, Dr. Henry Butler Allen. He has done a fine job in superImposing upon our regular Institute and Museum activities the organizing and directing of the war work which has been and is being done here. His value to the war effort of our country has been recognized by the National Defense Research Committee, commonly known as the N.D.R.C., by the Ordnance Department of the United States Army, and by other government agencies. In carrying on this extra work he has had the hearty backing of the Officers and Board of Managers of the Institute and of the staff, and I desire to take this opportunity of expressing our appreciation of his work, which has brought to the Institute the honor we are about to receive.

"It is our custom at meetings of the Institute to call upon Dr. Allen for a report and I am particularly happy today to continue that custom and to ask him to tell us something about the Institute's activities during these war years. You will, of course, realize that his remarks must be quite general, as most of the work which has been done here is of the character commonly termed 'hush-hush.'"

He then called upon the Secretary and Director, Dr. Henry Butler Allen. Dr. Allen spoke as follows:

"Mr. Redding, General Campbell, Members of the Board, Officers, Employees, Members and Friends of The Franklin Institute:

"I thank you, Mr. President, for myself and for all of us for your kind words; and for the enthusiastic support which you and the Board of Managers have given us in our efforts. Such expressions are a great satisfaction—particularly as coming from you, Sir.

"Heritage, if treated as a foundation upon which to build, and not as a completed structure in which to retire, can be real incentive for further achievement. We have this in The Franklin Institute—service to our Country in peace and in five wars—a record without a break since 1824.

"An early indication of preparedness influence is shown in our list of original members, among whom are recorded one E. I. duPont, powder manufacturer, and James Barron, Captain, U. S. Navy.

"We might even give credit to our patron saint, for Benjamin Franklin noted after seeing the first balloon ascension that the discovery might, and I use his words, 'possibly give a new turn to human affairs. Convincing sovereigns of the folly of wars may perhaps be one effect of it; since it will be impracticable for the most potent of them to guard his dominions. Five thousand balloons, capable of raising two men each, could not cost more than five ships of the line; and where is the prince who can afford so to cover his country with troops for its defence as that ten thousand men descending from the clouds might not in many places do an infinite deal of mischief before a force could be brought together to repel them?' Witness our parachutists!

"The Institute for years has fostered preparedness in peace time by giving encouragement to inventors of war material. We find a medal going in 1840 to one William Jenks of Springfield, Mass. for 'An Improvement in Fire Arms,' to a Lt. Bradley A. Fiske in 1892 for an 'Electrical Range Finder.' In 1905 John Browning was given a medal for automatic weapons —we all know the now famous Browning machine gun with which our Ordnance Department today arms our infantry and aircraft, to the discouragement of our enemies.

"Our Lectures have included well-known ordnance names in recent years such as General Joseph Wheeler, General C. C. Williams, both predecessors of our present Chief of Ordnance; Robert H. Kent, in whom General Campbell has one of today's best ballisticians; Major Scott B. Ritchie, who contributed so much to ordnance industrial preparedness since the last war and who now, as Colonel, is one of the General's main-stays under General G. M. Barnes in developing new weapons. And our own associate, Hiram Ely, a Colonel in this war, who has done much in aircraft detection and anti-aircraft fire control.

"Our Journal has printed papers by these lecturers as well as others, such as recently one by Colonel L. E. Simon, Head of the Ballistic Research Laboratory of the Ordnance Proving Ground, whose current work in sampling methods for inspection is assisting General Campbell in his present responsibilities.

"Our Museum displays some of the earliest weapons made in New England under the now famous interchangeable parts method of manufacture, forerunner of the mass production system. We have from Frankford Arsenal the original Gill Loading Machine, the granddaddy of modern ammunition loading machines which nowadays are turning out billions of cartridges; of course we show modern automatic and other weapons which embody significant features or historical development.

"Apart from a patriotic motive, ordnance problems have always interested The Franklin Institute. They naturally would, the Institute's field of interest being in the physical sciences and mechanics. And the Institute by reason of itself being in close contact with the world of pure and applied science and industry is in a rather unique position to give service. For the length of time is usually long between the discoveries of a pure science research worker and the engineering application of his discovery. And war doesn't wait for customary peacetime schedules. The sooner a nation is able to put into use the findings of the research laboratory, the greater are its chances of victory—with the least sacrifice of its sons in battle.

"Our Board of Managers—all who could possibly do so are here today—includes leaders in manufacture, engineering and research. Our technical organization carrying on our war work consists of all the scientists, engineers, technicians, mechanics and instrument makers, both men and women, in our Philadelphia building, in addition to the scientists and others of our out-of-town Bartol and Biochemical Research Laboratories. Last, but not least, are Institute members who volunteer their consulting services. We have expanded our staff, of course, for the war, now having in round figures a total of 316 workers on war work alone, as compared with a total enrollment of 200 before the war. We hope to retain some of those taken on since war started for an expanded postwar industrial research and development program. And back of all these are our office workers who are playing their important part in the effort.

"I only wish we could tell you about some of the war projects at the Institute. But the war is still on and the ban of secrecy is naturally still in force.

"Our Chief of Ordnance of the Army is awake to preparedness needs. I have known him well and long enough personally to vouch for that.

"We all read reports from the enemy as well as from our own field commanders about the excellence of our Ordnance material. It is a distinct honor to have here today the responsible head of that great Army Corps. The Navy, too, can be proud of him, for he was graduated from the Naval Academy in 1909. He soon transferred to the Army in that highly technical branch, the Coast Artillery. It must be a satisfaction to him to have next served under Major General Leonard Wood in the first Plattsburg training camps. Later he pioneered in the engineering of self-propelled gun mounts, tanks, and in new production methods for artillery ammunition. In 1940 he was put in charge of the vast program for building munition plants and depots over the country. In 1942 he was charged with all ordnance production, and soon thereafter was made Chief. Captain in 1917; Major General in 1942—still a young man—I have the great pleasure of introducing our 16th Chief of Ordnance of the United States Army, Major General Levin H. Campbell."

Major General L. H. Campbell, Jr., Chief of Ordnance, United States Army, then delivered an exceedingly interesting paper entitled "Ordnance Research and Development for Peace." *

At the conclusion of his speech, General Campbell presented to The Franklin Institute, through its President, Mr. Charles S. Redding, the Ordnance Distinguished Service Award rom the Ordnance Department of the United States Army. The citation reads as follows:

> "Presented to The Franklin Institute in recognition of outstanding and meritorious engineering advisory services, in war and peace, for the development, manufacture and maintenance of Ordnance materiel."

The award was authorized July 20, 1944.

Mr. Redding accepted the Certificate as follows:

"General Campbell, on behalf of the Officers, the Board of Managers, and the members of The Franklin Institute, I gratefully accept this Distinguished Service Award. It will be displayed and preserved among our archives as very real evidence that The Franklin Institute has, during this war, continued its ideal of service which it has maintained during its existence of exactly one hundred and twenty years. In accepting this award, I pay tribute to the men and women who have worked tirelessly on the many problems which they have been asked to solve. Our regular scientific staff has been expanded from time to time as additional problems came to us for solution, and the zeal with which all have attacked their problems is evidenced by the results obtained. While they worked without any thought of the recognition accorded them by this award, they must individually have a very real feeling of pride in having had a part in the job.

"While we of The Franklin Institute are justly proud of what has been accomplished here in connection with our country's war effort, we fully realize that it is but a very small part in a stupendous undertaking. Our country, called by our enemies a decadent Republic, has astounded the world by its accomplishments since Pearl Harbor. To reverse the scriptural quotation, we have, as a united people, truly beaten our ploughshares into swords. With this backing, our Armed Services are well on the road to final victory and, General Campbell, speaking for this audience, and through you, may I express our thanks and appreciation for the wonderful job done by the Ordnance Department which you represent and also by all the other branches of our Army and Navy.

"We are sitting here today in the almost living presence of Benjamin Franklin. Were it possible for that statue to come to life, how proud he would be of this Institution which bears his name and how proud he would be of this Republic in the early life of which he played such an important part! Incidentally, what would he think of our modern electronic developments, particularly those used in the prosecution of the war, stemming, as they do, from his single fluid theory of electricity?

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* General Campbell's addr- ss in full appears on page 239 of this issue.

"The Franklin Institute, spurred on by this award, will continue to perform such services as may be assigned to it by your department or other government agencies engaged in leading us to victory. However, we are all looking toward the day when peace will again reign on earth, the day when we can all return to our normal pursuits—the day when again The Franklin Institute, through its staff here in Philadelphia, its Bartol Research Foundation, and its Biochemical Research Foundation, may bend its principal efforts toward making this world a better place in which to live rather than devising and designing instruments of destruction. Even then, General Campbell, as you have indicated, your work will go on in keeping our country prepared against another war which we sincerely hope will never come. I am sure the Officers and Board of Managers of the Institute will be fully in accord with the statement that our services, then as now, will continue to be at your disposal. Thank you again."

The meeting was adjourned by the playing of "America" by Lieutenant Marriner.

Immediately after this special meeting, a reception and tea in honor of General Campbell, given by the Hostess Committee of The Franklin Institute, was held in the Graphic Arts section of the Museum. About four hundred distinguished guests, members and employees attended.

HENRY BUTLER ALLEN,
Secretary.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

The library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from nine o'clock A.M. until five o'clock P.M., Wednesdays and Thursdays from two until ten o'clock P.M.

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NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

Concentration of the Virus of Poliomyelitis by Differential Centrifugation.—D. W. McKinstry, H. M. Terrill, E. H. Reading and K. V. Huffman. The virus of tobacco mosaic disease was shown by W. M. Stanley in 1935 (*Science*, **81**: 644, 1935) to be essentially a crystalline nucleoprotein macromolecule. This finding provided a great stimulus to research designed to elucidate the fundamental nature of these agents of disease.

It is evident that progress in this phase of virology as well as the development of potent and non-toxic virus vaccines is dependent on the evolution of methods for the isolation, concentration and purification of the parasite without concomitant denaturation. Although almost phenomenal progress has been made in this connection during the past nine years with plant viruses, attempts at purification of the more labile animal viruses have been attended with little success. In contrast to the plant viruses, the latter are usually present in very low concentrations in the cells of the host and frequently are strikingly similar in physical and physico-chemical deportment to certain of the normal protoplasmic constituents. Furthermore, the classical methods of differential precipitation and adsorption which have been employed so successfully in the purification and crystallization of enzymes, usually have yielded preparations with distorted physical properties as well as marked loss of infectivity. Consequently, the attention of most investigators interested in this line of research has been turned toward the less drastic method of differential centrifugation.

An excellent review of the literature on the numerous attempts to purify the virus of poliomyelitis is given by Gard (Gard, Sven, "Purification of the Poliomyelitis Viruses," Almqvist and Wiksells Boktryskeri, Uppsala, Sweden, 1943). Unfortunately, in most instances, the starting material was subjected to a preliminary chemical treatment in an effort to remove a large percentage of the normal tissue components. Invariably this procedure has been accompanied by varying degrees of inactivation of the virus. It is obvious that although only a slight degree of denaturation of the virus complex results from the purification process such preparations do not represent a homogeneous system. None of the poliomyelitis preparations thus far reported has satisfied the accepted criteria of purity of macromolecules of biological origin.

The present investigation was undertaken with two objectives in mind: (1) The preparation of a small amount of pure, active virus for determination of the chemical nature and physical properties of the parasite and (2) concentration and partial purification of the agent for

the preparation of a concentrated vaccine by a procedure previously published (McKinstry, D. W. and Reading, E. H., *Journal of the Franklin Institute*, **237**: 71, 1944). Unfortunately, it was necessary to discontinue this study before either of these objectives was attained. However, certain of the data obtained seemed of sufficient importance to warrant publication of this preliminary communication.

MATERIALS AND METHODS.

1. *Strain of Virus.* The SK murine strain of the virus of poliomyelitis was used throughout this investigation. This strain, which is

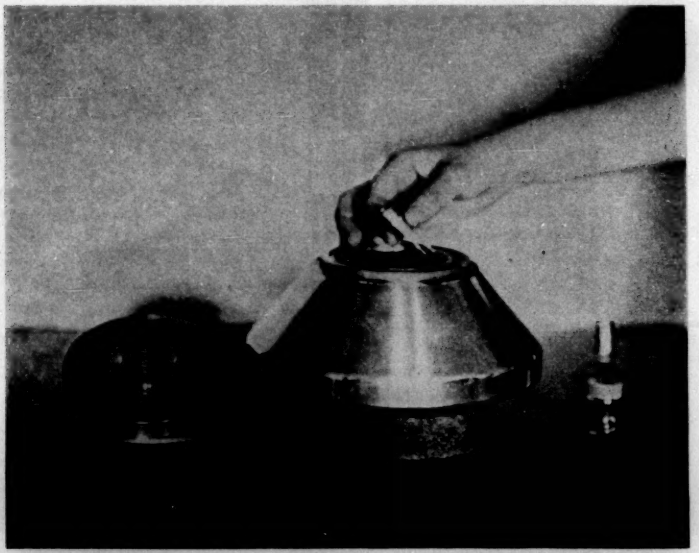


FIG. 1. Loading the tubes in the rotor.

highly virulent for mice, produces a typical infection characterized by flaccid paralysis of the extremities which rapidly progresses to a fatal end. Maximal virulence was maintained by repeated animal passage.

2. *Experimental Animals.* Swiss albino mice, both males and females, were used. They were usually held in the laboratory under standard conditions with a balanced diet several days before use. Preliminary studies indicated that these animals were uniformly susceptible to the strain of virus employed and that the experimental

infection ran a true clinical course. All of the mice used were obtained from a single source.

3. *Extraction of Virus.* Infected brain tissue aseptically removed from mice sacrificed shortly after the onset of paresis was quickly frozen with solid CO_2 and stored at $-70°$ C. for several weeks. This procedure was followed in view of the fact that it has been demonstrated

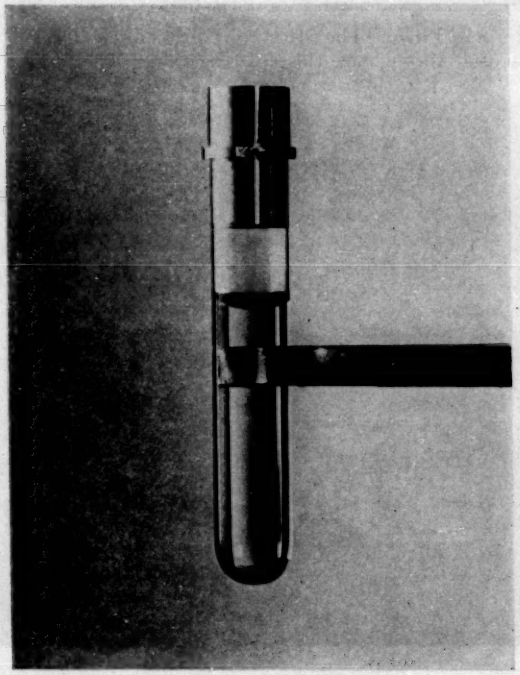


FIG. 2. Tube after centrifuging showing sediment and clear supernatant liquid.

(Loring, H. S. and Schwerdt, C. E., *Journal of Experimental Medicine*, **75**: 395, 1942) that the amount of sedimentable normal protoplasmic nitrogen decreases and becomes almost negligible after the tissue is held in the frozen state for several weeks. A large number of brains were usually pooled for each experiment. The tissue was homogenized at low temperature with sufficient sterile 0.1 per cent. glycine to make a 10 per cent. suspension by means of an apparatus especially

designed for the purpose (Terrill, H. M. and McKinstry, D. W., *Journal of the Franklin Institute*, **237**: 315, 1944). After refrigeration overnight at 4° C., cell debris was removed from the milky suspension by centrifugation at 1,500 r.p.m. for 10 minutes. Gross particulate

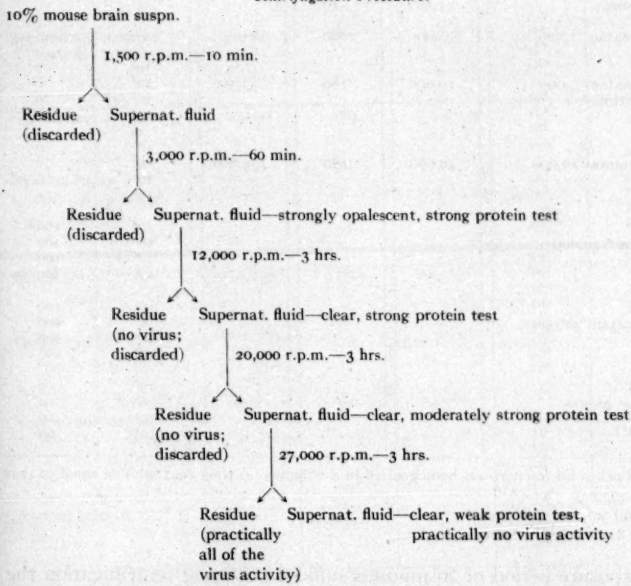
TABLE I.

Strength of the Centrifugal Field at Various Speeds of the Rotor.

Speed of Rotor (r.p.m.)	X Gravity
12,000	12,100
20,000	33,500
27,000	61,100
30,000	75,500
40,000	134,000

TABLE II.

Centrifugation Procedure.


10% mouse brain suspn.
1,500 r.p.m.—10 min.
Residue (discarded)
Supernat. fluid
3,000 r.p.m.—60 min.
Residue (discarded)
Supernat. fluid—strongly opalescent, strong protein test
12,000 r.p.m.—3 hrs.
Residue (no virus; discarded)
Supernat. fluid—clear, strong protein test
20,000 r.p.m.—3 hrs.
Residue (no virus; discarded)
Supernat. fluid—clear, moderately strong protein test
27,000 r.p.m.—3 hrs.
Residue (practically all of the virus activity)
Supernat. fluid—clear, weak protein test, practically no virus activity

matter remaining in the supernatant fluid was sedimented in the angle centrifuge at 3,000 r.p.m. for 60 minutes. The pink, strongly opalescent fluid was then carefully withdrawn for differential ultracentrifugation.

4. *Centrifugation.* An air-driven centrifuge of the Beams type with controls for accurately measuring and regulating the speed was used.

The material was held in celluloid tubes, about 13 × 70 mm., which were fitted snugly into cells in the aluminum alloy rotor at an angle of 45° to the axis of rotation. The mean distance from the axis was 75 mm. The capacity of the rotor was 16 tubes of 6 ml. volume each. The tubes were sterilized, after capping with cellophane, by exposure to ultraviolet radiation from a Westinghouse H-3 mercury arc lamp in a horizontal position at a distance of 3 inches from the outer envelope.

TABLE III.

Differential Centrifugation of SK Murine Poliomyelitis Virus.

Experiment 1.

Fraction	Treatment			Activity Test	
	Speed (r.p.m.)	Time (min.)	X Gravity	Dil. Virus †	Survivors % ‡
10% suspn.	1,500	10	—	—	—
Supernatant 1,500	3,000	60	—	—	—
Supernatant 3,000 (control)	20,000	180	33,500	10^2	0
				10^3	0
				10^4	0
				10^5	50
Supernatant 20,000	40,000	180	134,000	10^2	0
				10^3	0
				10^4	0
				10^5	75
Residue * 20,000	—	—	—	10^2	100
				10^3	100
				10^4	100
				10^5	100
Supernatant 40,000	—	—	—	10^2	100
				10^3	100
				10^4	100
				10^5	100
Residue 40,000	—	—	—	10^2	0
				10^3	0
				10^4	0
				10^5	60

* Each solid fraction was homogenized in a volume of 0.1 per cent. glycine equal to that of the original.

† 0.1 ml. intraperitoneally.

‡ Observation period of eight days.

An exposure period of 20 minutes sufficed. During centrifugation the tubes were closed and protected from contamination by means of aluminum alloy plugs which were originally designed to prevent the tubes from collapsing at high speed (Terrill, H. M., *Journal of the Franklin Institute*, **237**: 73, 1944). The plugs were sterilized by autoclaving before use. Fig. 1 shows the tubes being loaded in the rotor and Fig. 2 the tube after centrifuging. Before each run the rotor was

chilled to 4° C. At the end of a run of 3 hours' duration, the temperature would usually not exceed 12° C. The strength of the centrifugal field at various speeds of the rotor is given in Table I. The centrifugation procedure is represented schematically in Table II.

5. *Activity Tests.* All fractions were titrated for virus activity by

TABLE IV.

Differential Centrifugation of Poliomyelitis Virus.

Experiment 5.

Fraction	Treatment			Activity Test	
	Speed (r.p.m.)	Time (min.)	X Gravity	Dil. Virus	Survivors %
10% suspn.	1,500	10	—	—	—
Supernatant 1,500	3,000	60	—	—	—
Supernatant 3,000 (control)	20,000	180	35,500	10^2	0
				10^3	0
				10^4	0
				10^5	60
Supernatant 20,000	27,000	180	—	10^2	0
				10^3	0
				10^4	0
				10^5	75
Residue 20,000	—	—	—	10^2	100
				10^3	100
				10^4	100
				10^5	100
Supernatant 27,000	40,000	180	134,000	10^2	100
				10^3	100
				10^4	100
				10^5	100
Residue 27,000	—	—	—	10^2	0
				10^3	0
				10^4	0
				10^5	50
Supernatant 40,000	—	—	—	10^2	100
				10^3	100
				10^4	100
				10^5	100
Residue 40,000	—	—	—	10^2	100
				10^3	100
				10^4	100
				10^5	100

injecting 0.1 ml. amounts of serial dilutions of the preparation into Swiss albino mice via the peritoneum. The animals were then observed for a period of eight days. The final volume of the various fractions was always adjusted in such a manner that the concentration of virus (assuming all to be present) would be exactly the same as in the starting material. In this way the activity of the various fractions could be

compared in terms of M.L.D. and the degree of concentration expressed as the denominator of the dilution factor.

RESULTS.

The results of several typical experiments are given in Tables III and IV.

In most of the earlier experiments, the supernatant fluid from the 3,000 r.p.m. run was centrifuged at 20,000 r.p.m. for 3 hours in an effort to remove the heavier normal tissue components. The virus was then separated from this partially purified extract by centrifugation at 40,000 r.p.m. for 2 hours. In the course of various test runs, it became apparent that a speed of 27,000 r.p.m. was about the minimum that sufficed to sediment the virus completely. This does not imply, however, that it is the optimum speed for separation of the virus. There are some indications that a somewhat higher speed for a shorter period might be preferable. Further work on this point is desirable. Although the 2-hour centrifugation at 20,000 r.p.m. removed most of the extraneous material without separation of the virus, a small, but constant, amount of tissue proteinaceous material always separated with the virus in the final centrifugation. Re-centrifugation of the final preparation, interpolation of runs at various intermediate speeds and profile sampling of the various liquid fractions failed to eliminate the impurity. Electrophoretic analysis of the final preparation indicated the presence of three components with seven boundaries. Virus was present in varying amounts in all of the fractions.

Summary. The SK murine strain of the virus of poliomyelitis can be completely sedimented, without inactivation, from brain tissue homogenized in 0.1 per cent. glycine, by a centrifugal force 61,100 times that of gravity.

Preliminary centrifugal treatment at 1,500, 3,000, 12,000 and 20,000 r.p.m. for varying periods of time eliminates a large percentage of the normal protoplasmic constituents without loss of virus activity.

The purest preparations contained a small, but constant, amount of tissue proteinaceous material. Electrophoretic analysis revealed the presence of three components.

BOOK REVIEWS.

MODERN TURBINES, by Louis E. Newman, Allen Keller, John M. Lyons and Lawrence B. Wales. 175 pages, photographs and charts, 15 × 22 cms. New York, John Wiley & Son, Inc., London, Chapman & Hall, Ltd., 1944. Price $2.75.

Modern industrial turbines are custom-built to meet specific operating conditions differing so widely from each other that rarely are identical turbines built for different purchasers. This is particularly true in the field embracing turbine generators rated from 500 to 7500 kw, and including almost every conceivable variation in type and operating conditions. Initial steam pressures, for example, range from atmospheric pressure to more than 2000 lb. per square inch, and this diversity of operating steam pressure, coupled with other variables, results in a different design for almost every application. The proper application therefore is a specialty in which a knowledge of design plays an important part. But many practicing engineers at one time or another are confronted with the problem of turbine application, who do not have this specialized knowledge. Sometimes the result is a mis-application. The book at hand was written for just such cases.

The contents of the book was first published as a series of articles in the magazine, *Power Plant Engineering*. The articles have been edited, revised, and rearranged for this book. The work starts out with a description of the types of steam turbines and the range of conditions encountered, then the steam chart and theoretical steam rate are discussed. A large part of the book is devoted to estimating the performance of turbines in terms of the steam rate and in terms of the heat rate, and this is followed through by the treatment on the reading and interpretation of performance charts. At this point the reader has become familiar with the many factors to be considered in applying the right turbine to a job. Just what would happen affecting the price of the machine, the performance guarantees, physical dimensions, weight, etc., if any of the factors were changed is discussed. This practical consideration is well covered. The latter part of the book treats on steam turbine governors and generators. Appendixes give much useful information and there is a subject index.

Written by members of the General Electric Company turbine division, this book fills a need by engineers who are interested in the selection of the right turbine for the right job, but who do not wish to concern themselves with the intricacies of turbine design or details of turbine maintenance.

R. H. OPPERMANN.

WOOD CHEMISTRY, edited by Louis E. Wise. 900 pages, tables and illustrations, 16 × 24 cms. New York, Reinhold Publishing Corp., 1944. Price $11.50.

Some of the fundamental investigations in recent years which have marked the progress made in this subject include the clarification and elucidation of the chemistry and fine structure of cellulose; the development of the field dealing with surface properties of cellulosic materials; studies of extraneous components of wood, of the chemistry of lignin, and in wood analysis; the advances made in the studies of the hemicelluloses; progress in the chemistry of the cell wall to which the chemical microscopists have contributed signally. Lignin and its derivatives are among the world's greatest industrial wastes and recognition of this fact serves as a constant challenge to chemists, technologists and inventors. In view of the progress made and the challenge, this American Chemical Society Monograph is a timely contribution to the opening of new avenues of research.

The volume consists of writings by fourteen authors, under a single editorship. It first presents a treatment on the growth, anatomy and physical properties of wood, then it covers the phases of development in knowledge up to the present time. The chemical and physical evidence regarding the constitution of cellulose is given and there are treatments on cellulose

solvents, compounds and derivatives. This occupies a considerable part of the volume. More brief but of import are the sections devoted to extraneous components or those substances which are soluble in neutral solvents, and the surface properties of cellulose and wood which have been neglected until recently. The section on the chemical analysis of wood is a survey of chronological events and an exposition of methods for different determinations. The last division of the volume is on wood as an industrial raw material. It treats on the combustion of wood, the thermal decomposition, the delignification of wood, the chemical behavior of wood involving the industrial pulping processes and others, decomposition of cellulose and hemicelluloses by micro-organisms, and decomposition of lignin.

Every chapter and every section is a finished work. Arranged in logical order and well connected, this work represents the results of an exhaustive search of the literature, skillful digestion, and expert knowledge of the subject. It is of great value to researchers in this field.

R. H. OPPERMANN.

INTRODUCTORY ASTRONOMY, by J. B. Sidgwick. 137 pages, illustrations, 16 × 23 cms. New York, Philosophical Library, 1944. Price $2.50.

The beauty, mystery, and eternity of the heavens is being appreciated by an ever greater number of people. One of the reasons for this is the publication of books for the layman which are interesting as well as instructive and which show that astronomy is not an abstruse and learned subject reserved to the scientist and mathematician, but is available to the average man to understand and enjoy. The book at hand is just one of these. It is small and compact (137 pages) and is written with an exceptional insight into the situation of the layman. Actually it is divided into two parts. The first is mainly descriptive. It starts with the description of how the beginner finds his bearings in the observations of a starlit night. This leads to how to find and recognize the different constellations and a fascinating story of modern knowledge regarding the planets, moon, stars, comets and meteors, star clusters and nebulae. The second part is devoted to descriptions of the individual constellations, how to find them and the many objects of special interest for observers with binoculars. These are illustrated by forty-seven star maps. A most useful feature of these star maps, and a unique one, is the scale that is marked on each one. This scale represents a six inch ruler held at arm's length against the sky, which subtends approximately 13 degrees. There is a glossary of terms and a subject index. The book is a good introduction for the layman to the wonders and fascination of the night sky.

R. H. OPPERMANN.

PUBLICATIONS RECEIVED.

Light, Vision and Seeing, by Matthew Luckiesh. 323 pages, photographs and illustrations, 15 × 22 cms. New York, D. Van Nostrand Company, Inc., 1944. Price $4.50.

Dictionary of Organic Compounds, revised and edited by I. M. Heilbron. 1072 pages, 18 × 24 cms. New York, Oxford University Press, 1943. Price $30.00. Supplements to Volumes II and III. Price $1.00 each.

Thermodynamic Charts, by Frank O. Ellenwood and Charles O. Mackey. (Second edition.) 46 pages, 28 × 21 cms. New York, John Wiley & Son, Inc., 1944. Price $2.75.